Solving one-step equations

When solving an equation, you are trying to find the value(s) for the variable(s) in the equation that makes the equation true. To do this you must isolate (get alone) the variable on one side of the equal sign. There should be a number remaining on the other side. Then you will know what the value of the variable is.

To isolate a variable we will need to use inverse operations.

> ***Think Back***
>
> *Inverse operations can be found in Module 4. They are opposite operations of what is happening to the variable.*

> ***Problem Solving Tip***
>
> *The easiest way to remember the inverse operation is to "undo" what is being done to it. For example the opposite of addition is subtraction. Keep in mind though, that equations are "balanced" – equal on both sides. What you do to one side, you* **must** *do to the other.*

While viewing the next examples, see if you can derive an algorithm for solving one-step equations.

Example 3

A. Solve: $x + 7 = 16$

Solution

Since x is being added to 7, the opposite is to subtract, but we want to move the numbers not the variable. So subtract 7 from the left side to get x alone, but what you do to one side, you have to do to the other. So it's easier to say, "Subtract 7 from both sides."

$$\begin{array}{r} x + \cancel{7} = 16 \\ \underline{-\cancel{7} \quad -7} \\ \boxed{x \quad = 9} \end{array}$$

the 7s on the left cancel since $+7 - 7 = 0$

Therefore the value of *x* that makes this equation true is 9.

Always perform a check by substituting the value you found for the variable:

$$(\quad)+7=16$$
$$(9)+7\stackrel{?}{=}16$$
$$16=16$$

Then you are all set; circle the answer as your last step.

B. Solve: $x-8=3$

Solution

Since *x* is being subtracted by 8, the opposite is to add. So add 8 to both sides.

$$\begin{aligned} x-\cancel{8} &= 3 \\ \cancel{+8} &\quad +8 \\ \hline \boxed{x} &= 11 \end{aligned}$$

$$\begin{aligned} (\quad)-8 &= 3 \\ \text{check: } (11)-8 &\stackrel{?}{=} 3 \\ 3 &= 3 \end{aligned}$$

Therefore the value of *x* that makes this equation true is 11. If for some reason the two numbers at the end are not the same, then you haven't found the right answer. Go back and try again.

For example:

If you had accidentally multiplied both sides by 8 thinking that would cancel the 8 on the left, you would have gotten *x* = 24.

Now perform the check:

$$\begin{aligned} (\quad)-8 &= 3 \\ (24)-8 &\stackrel{?}{=} 3 \\ 16 &\neq 3 \end{aligned}$$

Notice the two numbers at the end are not equal to each other. Therefore the answer that you found is incorrect. You'll need to go back and try again.

Example 4

> ***Think Back***
>
> *Coefficients go before the variables.*

A. Translate and solve: Ten times a number is 120.

Solution

Let: *n* = a number

$$10n = 120$$

n is being multiplied by 10, the opposite is to divide. So divide by 10 on both sides.

$$\frac{\not{1}0}{\not{1}0}n = \frac{120}{10}$$

$$\boxed{n = 12}$$

$$10(\ \ ) = 120$$

$$\text{check: } 10(12) \stackrel{?}{=} 120$$

$$120 = 120$$

Therefore the value of *n* that makes this equation true is 12. When you have written a "Let" statement it is often properly expected of you to go back into the "Let" statement and write in the correct answer, like this:

Let: *n* = a number = 12

B. Translate and solve: Two-thirds of *x* is thirty-two.

Solution

$$\frac{2}{3}x = 32$$

x is being multiplied by $\frac{2}{3}$, the opposite is to divide. So divide by $\frac{2}{3}$ on both sides.

Recall that you can't divide by fractions, so let the calculator help you. Make sure that you use the fraction key, or recall the division algorithm: "Copy, change, flip".

$$\frac{\frac{2}{3}x}{\frac{2}{3}} = \frac{32}{\frac{2}{3}}$$

$$\boxed{x = 48}$$

$$\frac{2}{3}(\ \ ) = 32$$

$$\text{check: } \frac{2}{3}(48) \stackrel{?}{=} 32$$

$$32 = 32$$

Therefore the value of *x* that makes this equation true is 48.

C. Translate and solve: If Mary's age is divided by 3 it would be equal to 12.

Solution

Let: a = Mary's age

$$\frac{a}{3}=12$$

a is being divided by 3, the opposite is to multiply. So multiply by 3 on both sides.

$$(3)\frac{a}{3}=12(3)$$

$$\boxed{a = 36}$$

$$\frac{(\ \ )}{3}=12$$

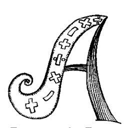
check: $\frac{(36)}{3}\overset{?}{=}12$

$$12=12$$

Therefore the value of n that makes this equation true is 36.

Let: a = Mary's age = 36

Did you come up with an algorithm for solving one-step equations? Here's a pretty good one:



Algorithm

Solving One-Step Equations:

1. Translate if necessary.
2. Get the variable alone by inverse operation, "undo it".
3. Check.

3. Translate (if necessary) and solve:

a. $x + \frac{1}{2} = \frac{1}{4}$

b. A number subtracted by 5.5 is 7.

c. $\frac{6}{7}n = -\frac{3}{8}$

d. $2.5y = 17.5$

Solving two-step equations

Notice in the last section it only took one step to undo the operation and solve for the value of the variable. Take a guess at how many steps it's going to take to solve these equations – you got it: 2! Although now it's a little more important to make sure that you follow some sort of an order to isolate the variable.

Here is an algorithm that should be able to aid you in solving these types of problems.

Algorithm

Solving Two-Step Equations:

1. Translate if necessary.
2. Get the variable alone by performing 2 inverse operations, SADMEP.
3. Check the answer.

Problem Solving Tip

The easiest way to think about the order of the 2 inverse operations is by performing PEMDAS backwards. Some have coined the phrase "SADMEP", its the order of operations backwards.

You want to undo addition/subtraction first and then undo multiplication/division next.

Example 5

A. Solve: $3x + 8 = 26$

Solution

The ultimate goal is to get *x* alone. Ask yourself: "If I knew what *x* was, what would I do first?" Hopefully you said, multiply it by 3 and then add it to 8. So go backwards undoing those operations, subtract 8 from both sides and then divide both sides by 3.

It should look like this:

$$\begin{array}{rcl} 3x+8 & = & 26 \\ -8 & & -8 \\ \hline 3x & = & 18 \end{array}$$

then:

$$\frac{3x}{3} = \frac{18}{3}$$
$$\boxed{x = 6}$$

check:

$$\begin{aligned} 3(\ \ )+8 &= 26 \\ 3(6)+8 &\stackrel{?}{=} 26 \\ 18+8 &\stackrel{?}{=} 26 \\ 26 &= 26 \end{aligned}$$

Therefore the value of *x* that makes this equation true is 6.

B. Translate and solve: Nine less than half of Ron's age is 11.

Solution

Let: *r* = Ron's age

$$\frac{r}{2}-9=11$$

Remember SADMEP, undo subtraction of 9 first and then undo the division of 2.

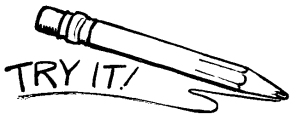
$$\begin{array}{rl} \frac{r}{2}-9=11 & \\ +9 \quad +9 & \leftarrow \text{Step 1: add 9 to both sides} \\ \hline (2)\frac{r}{2} = 20(2) & \leftarrow \text{Step 2: multiply both sides by 2} \end{array}$$
$$\boxed{r = 40}$$

check:

$$\begin{aligned} \frac{(\ \ )}{2}-9 &= 11 \\ \frac{(40)}{2}-9 &\stackrel{?}{=} 11 \\ 20-9 &\stackrel{?}{=} 11 \\ 11 &= 11 \end{aligned}$$

When you practice enough, you'll be able to do each step, one right after the other like this.

Therefore the value of *r* that makes this equation true is 40. Thus Ron's age is 40.



4. Translate the following and solve:

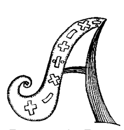
a. Twenty-four equals the quotient of *x* and -3 added to 4.

b. $5y - 7 = 33$

Solving Complex Equations: Combining like terms

Notice in the last section it took two steps to undo the operations and solve for the value of the variable. With a title like "complex equation," you know they are going to be a little more involved than that.

Here is an algorithm that should be able to aid you in solving these types of problems.

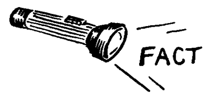


Algorithm

Solving Complex Equations - combining like terms:

1. Combine like terms.
2. Get the variable alone by performing inverse operations, SADMEP if possible.
3. Check the answer in the original equation.

Combine like terms means to group together terms that are the same (numbers with numbers / variables with variables) and are on the same side of the equal sign.



Like terms are 5x and -3x or -27y and 6y.
Their coefficients might be different but they have the same variables
-19xy and -19z are not like terms. Even though they have the same coefficient, they don't have the same variables.

Example 6

A. Solve: $4a+3a=56$

Solution

Notice that there are two sets of *a*'s on the left-hand side of the equation. You should be asking yourself, "How many *a*'s altogether?" There are 7 *a*'s, not 2. Thus:

$$4a+3a = 56$$

$$\frac{7a}{7} = \frac{56}{7} \quad \leftarrow \text{Divide both sides by 7.}$$

$$a = 8$$

check:

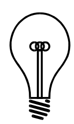
$$4(\ \ )+3(\ \ ) = 56$$

$$4(8)+3(8) \stackrel{?}{=} 56$$

$$32+24 \stackrel{?}{=} 56$$

$$56 = 56$$

Therefore the value of *a* that makes this equation true is 8.



Problem Solving Tip

A good way to think about combining like terms is to make a phrase to go along with it. For example, in the last problem, $4a+3a=56$*. 4 "apples" plus 3 "apples" is a total of 7 "apples" and that's equal to 56.*

B. Solve: $2p-6p=40$

Solution

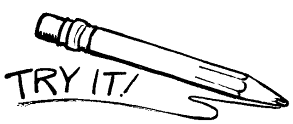
Notice that there are two sets of p's on the left-hand side of the equation. You should be asking yourself, "How many p's altogether?" "I have 2 'plums' and I take away 6 'plums' then I have -4 'plums' left." Thus:

$$\begin{aligned} 2p-6p &= 40 \\ \frac{-4p}{-4} &= \frac{40}{-4} \quad \leftarrow \text{Divide both sides by } -4. \\ p &= -10 \end{aligned}$$

check:

$$\begin{aligned} 2(\ \ )-6(\ \ ) &= 40 \\ 2(-10)-6(-10) &\stackrel{?}{=} 40 \\ -20--60 &\stackrel{?}{=} 40 \\ -20+60 &\stackrel{?}{=} 40 \\ 40 &= 40 \end{aligned}$$

Therefore the value of p that makes this equation true is -10.



5. Solve: $-5c+9c=-20$.