Lesson 6

Two-Step Equations

Objective

⇨ Solve two-step equations of the form:

- $ax + bx = c$
- $ax + b = c$
- $ax + b = cx + d$

As you have learned, solving equations involves writing equivalent equations until you have an equation of the form $x = c$ where c is a constant. The problems you have solved thus far can be done in one step. Some equations require more than one step to solve.

Example 1

Solve the equation $5x + 6x = -22$ for x.

Solution

The left side of the equation contains like terms.

You may write an equivalent equation by combining the like terms.

$$5x + 6x = -22$$

$$11x = -22 \qquad \text{Combine like terms}$$

This new equivalent equation is a one-step equation that you can solve.

$$11x = -22$$

$$\frac{11x}{11} = \frac{-22}{11} \qquad \text{Division Property of Equality}$$

$$x = -2$$

Check:

$$5x + 6x = -22$$
$$5(-2) + 6(-2) = -22$$
$$-10 - 12 = -22$$
$$-22 = -22$$

Algorithm

To solve equations of the form $ax + bx = c$ **:**

1. Combine like terms.
2. Solve the resulting one-step equation.

Example 2

Solve the equation $2b - 3b + 8b = -21$ for b.

Solution

$$2b - 3b + 8b = -21$$

$$7b = -21 \qquad \text{Combine like terms}$$

$$\frac{7b}{7} = \frac{-21}{7} \qquad \text{Division Property of Equality}$$

$$b = -3$$

Solve each equation below. Name the property that you use in each step to form an equivalent equation.

1. $-3y + 6y = 9$

2. $8x + 9x - 4x = 39$

Example 3

Solve the equation $3x - 6 = 12$ for x.

Solution

In this equation the variable, x, has been multiplied by 3 and then 6 was subtracted to get 12. To solve this equation for x, undo this process in the exact reverse order. Starting with 12, undo the subtraction by adding 6. Then undo the multiplication by dividing by 3.

$$\begin{aligned} 3x - 6 &= 12 \\ +6 \quad &+6 \quad \text{Addition Property of Equality} \\ 3x &= 18 \\ \frac{3x}{3} &= \frac{18}{3} \quad \text{Division Property of Equality} \\ x &= 6 \end{aligned}$$

Algorithm

To solve an equation of the form $ax + b = c$:

1. Add the opposite of b to both sides of the equation.
2. Divide both sides of the equation by a.

Example 4

Solve the equation $3y - 4 = -1$ for y.

Solution

$$\begin{aligned} 3y - 4 &= -1 \\ +4 \quad &+4 \quad \text{Addition Property of Equality} \\ 3y &= 3 \\ \frac{3y}{3} &= \frac{3}{3} \quad \text{Division Property of Equality} \\ y &= 1 \end{aligned}$$

Problem Solving Tip

In order to "unravel" an equation, the unraveling must be done in the exact opposite order that the equation was built.

Solve each equation. Name the property that you use in each step.

3. $5x + 6 = 24$

4. $-x + 2 = 0$

Example 5

Solve the equation $5x + 6 = 3x - 2$ for x.

Solution

In the equation $5x + 6 = 3x - 2$, the like terms are on opposite sides of the equation.

In order to combine the like terms, use the Addition Property of Equality twice.

$$\begin{array}{rcl} 5x+6 & = & 3x-2 \\ \underline{\quad -6} & & \underline{\quad -6} \\ 5x & = & 3x-8 \\ \underline{-3x} & & \underline{-3x} \\ 2x & = & -8 \end{array}$$

Addition Property of Equality

Addition Property of Equality

Solve the resulting equation using the Division Property of Equality.

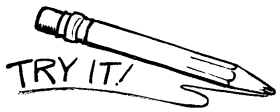
$$\frac{2x}{2} = \frac{-8}{2}$$
$$x = -4$$

Think Back

If you think of subtraction as adding the opposite, you can use the Addition Property of Equality if you add or subtract.

To solve equations of the form $ax + b = cx + d$:

1. Add the opposite of *cx* and the opposite of *b* to both sides of the equation.
2. Solve the resulting one-step equation.



5. Solve the equation $8x + 7 = 5x - 2$ for x.

6. Solve the equation $8x + 7 = 5x - 2$ by combining the *x*-variables on the right side of the equation and the constants on the left side. Does the equation result in the same solution for *x*?

Example 6

A bank charges \$3.00 per month as a service charge for a checking account. In addition, there is a \$0.02 charge per check. If an account was charged \$3.78 in one month, how many checks were written? Use an equation to solve.

Solution

The expression $0.02x$ represents the check charges for x checks.

If the service charge of \$3.00 is added to this value, the total is \$3.78.

The equation is $3.00 + 0.02x = 3.78$.

$$\begin{aligned} 3.00 + 0.02x &= 3.78 \\ -3.00 \quad & \quad -3.00 \qquad \text{Addition Property of Equality} \\ 0.02x &= 0.78 \\ \frac{0.02x}{0.02} &= \frac{0.78}{0.02} \qquad \text{Division Property of Equality} \\ x &= 39 \end{aligned}$$

Therefore 39 checks were written in one month.

7. Choose the equation that represents the following problem. Then solve the problem.

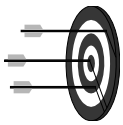
Keturah earns a base pay of $500/week. Her overtime pay is $9.00/hour.
If Keturah's paycheck before taxes was $644, how many hours of overtime did she work?
Let x = the number of hours of overtime Keturah worked.

a. $500 + 9x = 644$ b. $644 + 9x = 500$ c. $500x + 9x = 644$ d. $500(9 + x) = 644$

Review

1. Highlight the three algorithms in this lesson.

2. Write one new thing that you learned in this lesson or write a question that you would like to discuss with your mentor.



Practice Problems

Unit 3 Lesson 6

Directions: Write your answers in your math journal. Label this exercise Unit 3 – Lesson 6 Set A and Set B.

Set A

Solve each equation below and check each of your answers.

1. $4a + 5a = 18$
2. $-7b + 12b = 20$
3. $-8c - 3c = 121$
4. $3x - 1 = -16$
5. $3x - 1 = 2x + 5$
6. $5y - 2 = 6y + 8$
7. $7z - 5 + 3z = 25$
8. $3a - 5 = a + 9$

Set B

1. Raul has $46.50 to spend on 4 concert tickets and parking. If he must spend $5.50 for parking, what is the most that Raul can spend on each ticket? Use an equation to solve.
2. Write and solve an equation to determine the number of quarters that must be added to $1.37 to make $3.62.
3. Annette earns a salary of $152 per week, plus a commission of $12 for each painting she sells. How many paintings must she sell in one week to earn $440?

1. $-3y+6y=9$

$3y=9$ Combine like terms

$\frac{3y}{3}=\frac{9}{3}$ Division Property of Equality

$y=3$

2. $8x+9x-4x=39$

$13x=39$ Combine like terms

$\frac{13x}{13}=\frac{39}{13}$ Division Property of Equality

$x=3$

3. $5x+6=24$

$\underline{-6}\quad\underline{-6}$ Subtraction Property of Equality

$5x=18$

$\frac{5x}{5}=\frac{18}{5}$ Division Property of Equality

$x=\frac{18}{5}$

4. $-x+2=0$

$\underline{-2}\quad\underline{-2}$ Subtraction Property of Equality

$-x=-2$

$\frac{-x}{-1}=\frac{-2}{-1}$ Division Property of Equality

$x=2$

5. $8x+7=5x-2$

$\underline{-5x-7}\quad\underline{-5x-7}$ Addition Property of Equality

$3x=-9$

$\frac{3x}{3}=\frac{-9}{3}$ Division Property of Equality

$x=-3$

6. $8x+7 = 5x-2$

$\underline{-8x+2} \quad \underline{-8x+2}$ Addition Property of Equality

$9 = -3x$

$\frac{9}{-3} = \frac{-3x}{-3}$ Division Property of Equality

$-3 = x$

Yes, the equation results in the same solution as Try It 5.

7. $9x$ is the amount of overtime pay.

If this is added to the base pay, you get the amount of the paycheck before taxes.

Therefore the correct equation is $500+9x=644$ (choice a).

$500+9x = 644$

$\underline{-500} \quad \underline{-500}$ Subtraction Property of Equality

$9x = 144$

$\frac{9x}{9} = \frac{144}{9}$ Division Property of Equality

$x = 16$

Keturah worked 16 hours of overtime.

End of Lesson 6