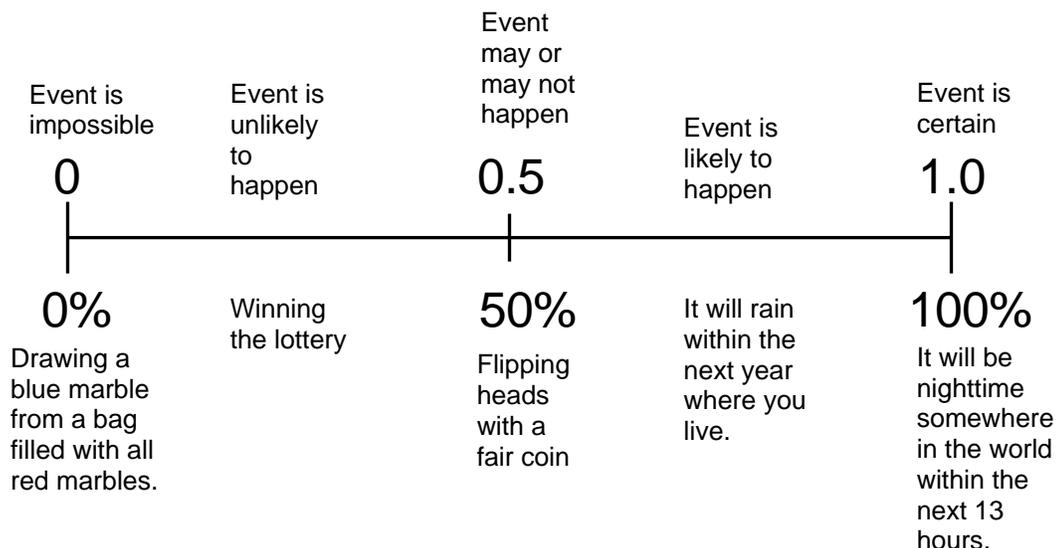# Lesson 4
An Introduction to Probability

Objectives

- Understand the concept of probability
- Understand that probabilities range from 0% to 100%
- Find the probability a particular outcome in a simple experiment

> **Probability** is the branch of mathematics that predicts how likely an event is going to occur.

Probability can be expressed as a fraction, decimal, or percent from 0% to 100%.

Event is impossible	Event is unlikely to happen	Event may or may not happen	Event is likely to happen	Event is certain
0		0.5		1.0
0%	Winning the lottery	50%	It will rain within the next year where you live.	100%
Drawing a blue marble from a bag filled with all red marbles.		Flipping heads with a fair coin		It will be nighttime somewhere in the world within the next 13 hours.

There are many applications of probability in science.

Example 1

Interpret the following probability statement.

There is an 80% chance of rain today.

> #### Solution
>
> This means that based on weather data, it rained at some point on 80% of the days that had today's weather conditions.

Probability is also used to predict the outcome of games.

Example 2

Interpret the following probability statement.

The chances of winning the lottery game "Crazy Larry's Cash Grab", are 1 in 200,000.

> #### Solution
>
> This means that if 200,000 tickets are printed, only one of them will be a winner. The fraction of winning tickets is $\dfrac{1}{200,000} = 0.000005 = 0.0005\%$.

TRY IT!

Write the question number on the number line to show the approximate likelihood of its corresponding event occurring.

1) Flipping tails on a fair coin

2) Drawing a red marble from a bag with one red marble and seven blue marbles

3) Flipping 1,000 heads in a row with a fair coin

4) Rolling a fair die, and getting a 1, 2, 3, 4, 5, or 6

5) A baby being born somewhere in the world in the next 10 minutes

```
|————————————————|————————————————|
0%               50%              100%
```
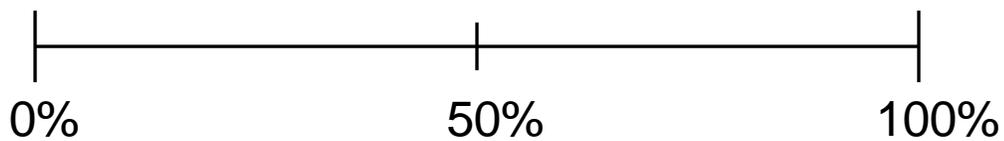
Write *true* or *false* next to each interpretation of the following probability statement.

The probability of drawing a blue marble from a bag of 50 marbles is 30%.

6) There are 30 blue marbles in the bag.

7) If a marble is drawn randomly and replaced in the bag, we expect blue to be drawn 30% of the time.

8) The probability of drawing a blue marble is $\dfrac{30}{100}$.

9) There is a 70% chance that the marble drawn will not be blue.

10) It is likely you will draw a blue marble from the bag.

Use the next paragraph to learn new vocabulary words.

"Today, we will do an **experiment** by picking marbles from a bag, one by one, and recording which one is drawn. The **sample space** of the bag of marbles is $\{3\,green, 2\,red, 1\,silver\}$. One possible **outcome** of this experiment is picking a green marble. One **event** we could find the probability of is drawing a red or a silver marble."
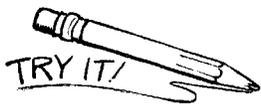
 Use the bolded terms from above to fill in the blanks below.

11) One marble is drawn from a bag containing 3 green marbles, 2 red marbles, and one silver marble. This is an example of a(n) _____.

12) Picking one silver marble is a(n) _____.

13) The sample space for this experiment is _____.
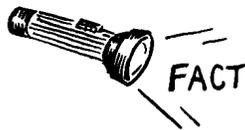
The probability of an event occurring is

$$P(\text{event}) = \frac{\text{number of desired outcomes}}{\text{total number of outcomes}}$$

Example 3

Sal is performing an experiment by randomly drawing a marble from a bag containing 2 orange, 3 yellow, and 4 green marbles. What is the probability that he draws a green marble? Write your answer to the nearest percent.

Problem-solving Tip
Model an experiment by sketching a picture.

Solution

We will use the formula.

$$P(\text{event}) = \frac{\text{number of desired outcomes}}{\text{total number of outcomes}}$$

We want to find the probability of drawing a green marble. We show this with the following notation.

$$P(\text{green}) =$$

or

$$P(g) =$$

Next, notice there are 4 chances at green…

$$P(g) = \frac{4}{}$$

…out of 9 total chances.

$$P(g) = \frac{4}{9}$$
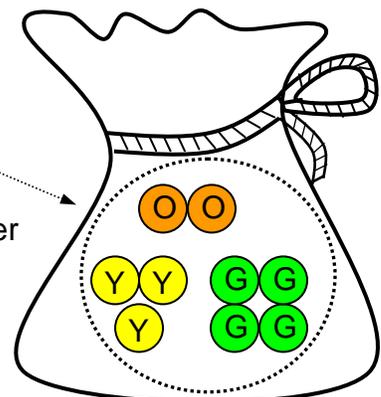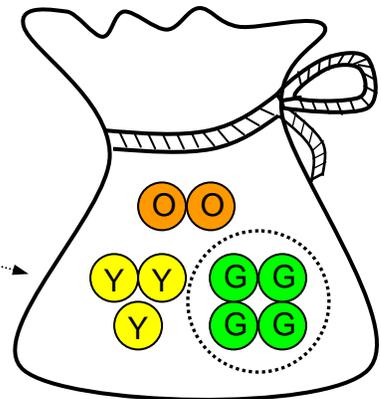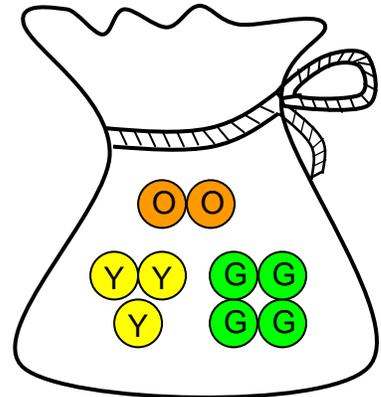
Finally, convert this probability to a decimal to answer The question.

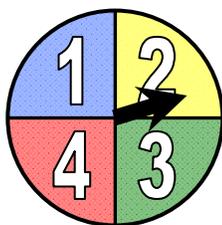$$P(g) = \frac{4}{9} = 0.44\overline{4} \approx 44\%$$

You roll a 12-sided die numbered 1 through 12. Rounded to the nearest percent, what is:
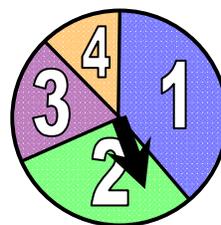
14) $P(7)$

15) $P(\text{even number})$

16) $P(13)$

The outcomes of an experiment are **equally likely** if each outcome has the same probability.

The outcomes of this spinner are **equally likely** (25%), since each section is the same size.

The sections of this spinner are of unequal size. Thus, the outcomes are **un**equally likely.

Example 4

Using the spinner at left, what is the probability you will spin:

a) 3

b) not 3

Solution

a) You should be able to see that the section marked **3** is one outcome out of four equally likely outcomes. Therefore,
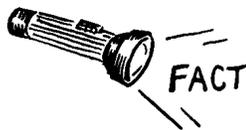
$$P(3) = \frac{1}{4}$$

b) To find the probability of not spinning a 3, means that you can spin a 1, 2, or 4. There are 3 sections out of four equally sized sections. Therefore,

$$P(\text{not } 3) = \frac{3}{4}$$

You may have noticed something interesting. The probability of spinning a 3 or not spinning a 3 is,

$$P(3 \text{ or not } 3) = \frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1 = 100\%$$

FACT

The probability of an event happening or not happening is always 1.

*An event not happening is called the **complement** of an event.*

If the probability of event A happening is $P(A)$, the probability of its complement is $1 - P(A)$

TRY IT!   A standard, six-sided, fair die is rolled.

17) $P(\text{lower than 3}) =$

18) $P(3 \text{ or higher}) =$

19) Are the events "lower than 3" and "3 or higher" complementary events? Explain your reasoning.

_____

_____

_____

_____

_____

☑Review

**Before you go on, be sure you understand these concepts:**

1. Probability is the mathematics of predicting the likelihood of uncertain events occurring.

2. Probabilities range from 0, impossible, to 1, absolutely certain.

3. Probabilities can be written as a fraction, decimal, or percent.

4. The probability of an event is given by the formula,

$$P(\text{event}) = \frac{\text{number of desired outcomes}}{\text{total number of outcomes}}$$

5. The probability of an event not occurring, as a decimal, is

$$P(\text{not event}) = 1 - P(\text{event})$$

Write one question you would like to ask your mentor, or write one thing you learned in this lesson.

_____

_____

_____

_____
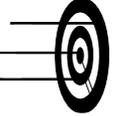
Practice Problems
Unit 4 Lesson 4

Directions: Write your answers in your math journal. Label this exercise
Unit 4 – Lesson 4, Set A and Set B.

Set A

You are looking at data about the languages students study in your school. Of the students in your school, 320 take French, 578 take Spanish, and 355 students do not take a language. Assume that no student studies more than one language.

1)   (T / F) It is likely that a student chosen at random takes Spanish.

2)   What is the likelihood that a student chosen at random studies French?

3)   What is the probability that a student chosen at random is taking a language?

4)   What is the probability that a student <u>who is taking a language</u> is taking Spanish?

5)   List a pair of events that are complementary.

6)   Verify that your answer to 5 is correct by adding the probability of each event. Show that it is equal to 1, or 100%.

Set B

1) Explain why the probability of an event cannot be greater than 1.

2) You are performing an experiment by randomly drawing one marble from a bag of equally-sized marbles. You record its color and replace it back in the bag. Then, you repeat the process. Suppose that the probability of drawing a green marble from the bag is 10%. Suppose that you draw 25,000 green marbles in a row. What is the probability of drawing a green marble on the 25,001st draw?

3) Suppose that out of a bag of equally sized marbles, there is one orange marble, and the probability of drawing it is 4%. What is the smallest possible number of total marbles in the bag in order for this to be true?

4) Give an example of an experiment with equally likely outcomes. Give an example of an event with unequally likely outcomes.

ANSWERS TO TRY IT

3)   2)   1)   4),5)

0%   50%   100%
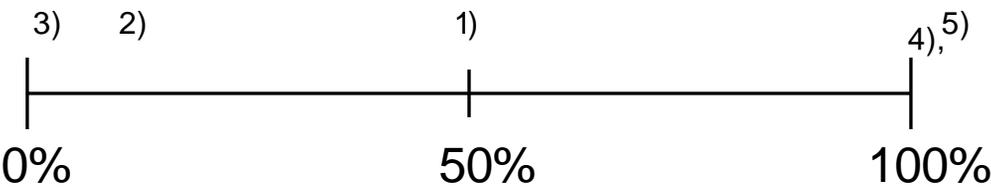
6) F   **7) T**   **8) T**   **9) T**   **10) F**

11) experiment   **12)** outcome or event   **13)** {green, red, silver}

14) $P(7) = \dfrac{1}{12} \approx 8\%$ **15)** $P(\text{even}) = \dfrac{1}{2} = 50\%$ **16)** $P(13) = 0\%$

17) $\dfrac{2}{6} = \dfrac{1}{3}$ **18)** $\dfrac{4}{6} = \dfrac{2}{3}$

19) Yes, these events are complementary since their probabilities add to 1.

$$\dfrac{1}{3} + \dfrac{2}{3} = 1$$

End of Lesson 4