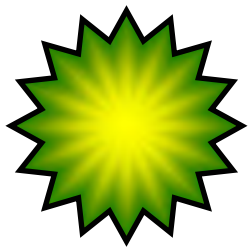
Lesson 7
Solving Equations: Part One

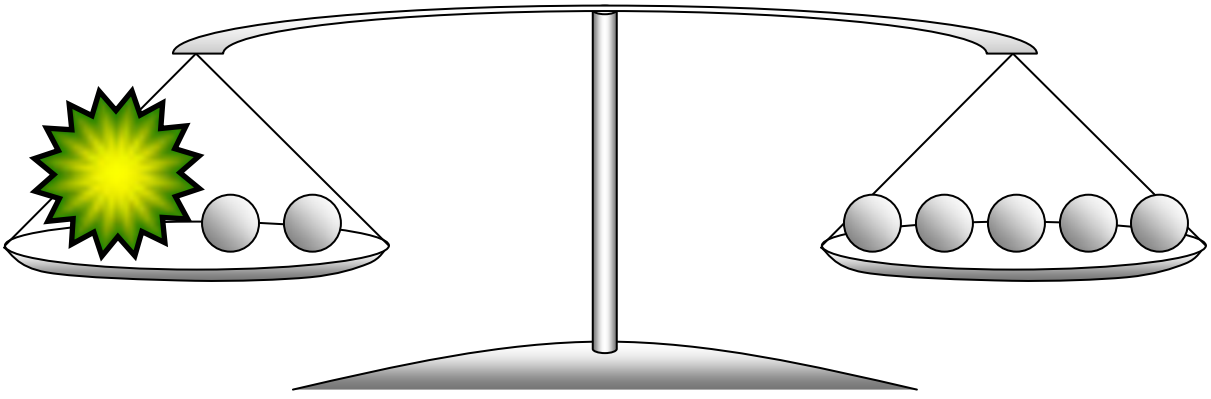
Objectives

- Given an equation, solve for a variable
- Check answer using substitution

You and your friend, Julio, are enjoying trading and playing with marbles. One morning, as you play down by the schoolyard, one of your marbles rolls into some bushes nearby. You begin searching for it, and you find something extraordinary. "Julio, come look at this!" you exclaim. "What is it?" Julio asks, as he runs toward you. You show him what you found.



It's a gemstone. Julio is jealous. "Dude, I will trade you for that," Julio says. You see how much your friend likes the gem. You decide to be a good friend, and trade fairly. "Okay Julio, but you have to make sure you pay me its weight in marbles." Julio agrees. With your scale, you balance the gem with marbles.



"How does this help us figure out how many marbles I owe you?" Julio asks. We can think about the weight of the gem in the following way:

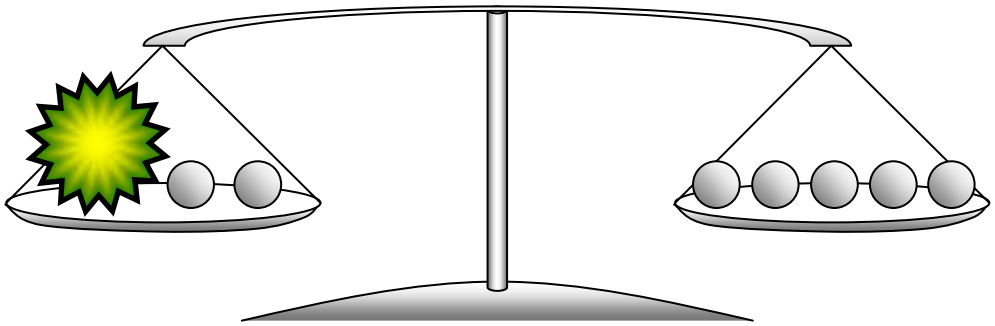
We know each marble weighs the same. Let's say the weight of one marble is 1.

Let  $= 1$

The weight of the gem could be anything. It's an <u>unknown</u>. Let's use a variable.

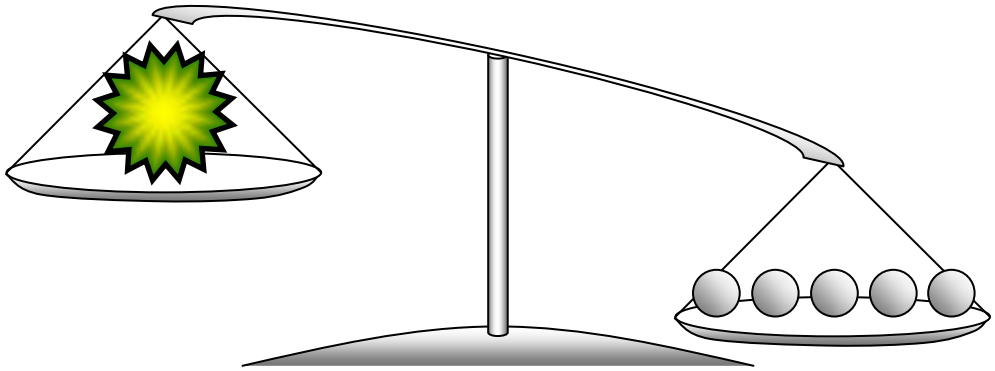
Let  $= x$

Now, we can use an <u>equation</u> to model our scales.



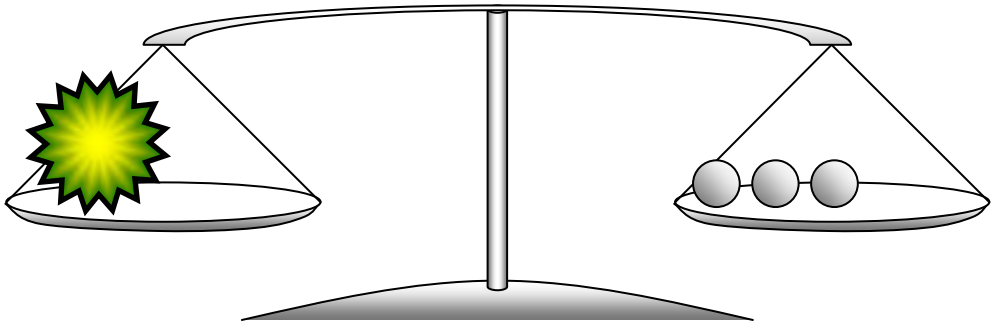
$$x + 2 \quad = \quad 5$$

In order to find its weight, the gem needs to be by itself, and the scales need to balance. You tell this to Julio, and he says, "That's easy! Just take two marbles away from the left pan!" After Julio does this, the scales do not balance.



$$x \quad \neq \quad 5$$

You think out loud to your friend. "Since we took two from the left pan, we have to take two from the right pan, as well."



$$x = 3$$

As the scales balance, you tell Julio, "That will be three marbles, please!"

Example

Solve for n. $n + 22 = 38$

Solution

$$\begin{array}{rcr} n + 22 & = & 38 \\ -22 & & -22 \\ n & = & 16 \end{array}$$

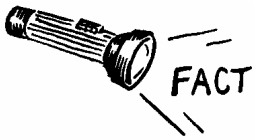
These cancel.

Start by drawing a vertical line on both sides of the equal sign. These represent the fulcrum of the balance scale.

The object is to get the variable, n, all by itself. Notice 22 is being added to n. We get rid of it by subtracting 22. But, in an equation, whatever operation is done on one side of the "=" sign must be done on the other side. We must also subtract 22 from 38, in order to keep the equation equal.

Notice that in both examples, one with the balance scales, and one with numbers, when a number was being added to our variable, we got rid of it by subtracting. This is because addition and subtraction are **inverse operations**.

- An **inverse operation** is an operation that undoes another operation.
  - Addition and subtraction are inverse operations.
  - Multiplication and division are inverse operations.



Another word for the inverse is opposite.

Addition is the opposite of subtraction.

Subtraction is the opposite of addition.

Multiplication is the opposite of division.

Division is the opposite of multiplication.

Example

Solve for h. $h - 13 = 4$

Solution

First, we will rewrite our equation with lines on either side of the equal sign.

$$h - 13 \mid = \mid 4$$

Now, whenever we write the "=" sign, we'll be sure to keep it between the lines.

Next, we will do the opposite of subtracting 13, which is adding 13. We will do this to both sides of the equation.

$$\begin{array}{r|c|l} h - 13 & = & 4 \\ +13 & & +13 \end{array}$$

The next step is to simplify both sides of the equation.

$$\begin{array}{rcl} h - 13 & = & 4 \\ +13 & & +13 \\ \hline & & +17 \end{array}$$

Lastly, we write our final answer.

$$\begin{array}{rcl} h - 13 & = & 4 \\ +13 & & +13 \\ \hline & & 17 \\ \\ h & = & 17 \end{array}$$

Believe it or not, we can always prove our answer is correct. We do so using substitution! To check our answer, we will substitute the number 17 for *h*.

Check: $h = 17$

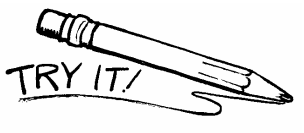
$$h - 13 \stackrel{?}{=} 4$$

$$(17) - 13 \stackrel{?}{=} 4$$

$$4 = 4 \ \checkmark$$

Problem Solving Tip

When reading a solution, use a piece of paper to cover up everything but the first line of the solution. Think about what the next step will be. Uncover the next line. See if you were correct. If not, think of where you went wrong. Continue this process for each line of the solution.



Solve for the variable in each equation. Then check to show your answer is correct.

1) $y + 5 = 9$	2) $x - 3 = 12$	3) $4 + a = 19$
4) $f + 25 = 57$	5) $c - 9 = 28$	6) $17 = k + 2$
7) $-3 = k + 19$	8) $42 = t - 2$	9) $r - 10 = 8$

Example

Solve the equation for k: $4k = 28$

Solution

Since the 4 and the k are being multiplied, we know we must <u>divide</u> both sides by 4 to get the k alone. We show this by making a fraction.

$$\frac{4k}{4} = \frac{28}{4}$$

Think Back

When numbers are next to variables, with no sign in between them, it means they are being multiplied.

$4k = 4 \cdot k$

Next, we see that the fours on the left hand side cancel (since $\frac{4}{4} = 1$), so the left hand side becomes k (since $1k = k$). On the right hand side, we divide 28 by 4, to preserve equality.

$$\begin{aligned} \frac{4k}{4} &= \frac{28}{4} \\ k &= 7 \end{aligned}$$

Finally, we check our answer.

Check: $k = 7$

$$\begin{aligned} 4k &\stackrel{?}{=} 28 \\ 4(7) &\stackrel{?}{=} 28 \\ 28 &= 28 \checkmark \end{aligned}$$

Example

Solve for w. $-8w = 104$

Solution

$$\begin{aligned} \frac{-8w}{-8} &= \frac{104}{-8} \\ w &= -13 \end{aligned}$$

Check: $w = -13$

$$\begin{aligned} -8w &\stackrel{?}{=} 104 \\ -8(\ ) &\stackrel{?}{=} 104 \\ -8(-13) &\stackrel{?}{=} 104 \\ 104 &= 104 \checkmark \end{aligned}$$

Think Back

- *When multiplying or dividing numbers with the same sign (+, +) or (-,-), the answer is positive.*
- *When multiplying or dividing numbers with different signs (+,-) or (-,+), the answer is negative.*

Example

Solve for p. $\frac{p}{4} = 6$

Solution

Since p is being divided by 4, multiply each side by 4.

$$4 \bullet \frac{p}{4} = 6 \bullet 4$$
$$p = 24$$

Check: $p = 24$

$$\frac{p}{4} \stackrel{?}{=} 6$$
$$\frac{(\ )}{4} \stackrel{?}{=} 6$$
$$\frac{(24)}{4} \stackrel{?}{=} 6$$
$$6 = 6 \checkmark$$

Solve for the variable in each equation. Then check to show your answer is correct.

10) $2a = 14$

11) $\frac{x}{3} = 11$

12) $-6s = 96$

13) $-9 = \frac{m}{4}$

14) $8y = -88$

15) $25z = 100$

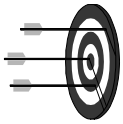
16) $\frac{h}{5} = 5$

17) $\frac{x}{4} = 8$

18) $-13t = 130$

Review

1. Highlight the definition of "inverse operation."
2. Highlight each "Think Back" and "Fact" box.
3. Write one question that you would like to ask your mentor, or one new thing you learned in this lesson.



Practice Problems

Unit 1 Lesson 7

Directions: Write your answers in your math journal. Label this exercise Unit 1 – Lesson 7, Set A and Set B.

Set A

Fill in the blanks to make each sentence true.

1) To solve the equation $x + 2 = 7$, the first step is to _____ from each side.

2) To check an answer, use a technique called _____.

Solve for the variable in each equation, and check your answer.

3) $4x = 12$ 4) $n + 1 = 7$ 5) $14 = z - 5$

6) $\frac{r}{6} = 7$ 7) $x - 80 = 120$ 8) $64 = 8w$

9) $z + 80 = 12$ 10) $y + 4 = -7$ 11) $\frac{t}{3} = -19$

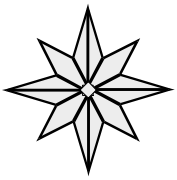
Set B

1) Solve the riddle algebraically. Use a let-statement. Create and solve an equation, and check your answer.

   Riddle: When a number is divided by six, it is equal to one and a half. What is the number?

2) Remember that we use inverse operations to undo something. For instance, in $3x$, we would undo the 3 by dividing. What would we do to change $\frac{2}{3}x$ into x?

1)	$y = 4$	10)	$a = 7$
2)	$x = 15$	11)	$x = 33$
3)	$a = 15$	12)	$s = -16$
4)	$f = 32$	13)	$m = -36$
5)	$c = 37$	14)	$y = -11$
6)	$k = 15$	15)	$z = 4$
7)	$k = -22$	16)	$h = 25$
8)	$t = 44$	17)	$x = 32$
9)	$r = 18$	18)	$t = -10$



End of Lesson 7