Lesson 7

Order of Operations

Objectives

- Understand the importance of defining a particular order of math operations
- Use the correct order of operations in arithmetic expressions
- Begin to understand fractions as division

When you read a book (like this one), you read the words from left to right, and from top to bottom. Why do you think this is? Couldn't we just as easily read from right to left, or from bottom to top? We could read upside down, or inside out, if we really wanted. Some ancient peoples read from left to right on one line, then from right to left on the next line. So why do we read the way we do?

The reason is that there is no reason! People just had to pick a way to read, and everyone in the culture had to agree. This way, people could write down their ideas, and everyone could understand what they meant, even after they were dead.

Mathematics, just like our language, is about communicating ideas. Part of its beauty is that ideas in math, written by someone over one thousand years ago, can be read and understood by people today. Just as with written language, mathematicians have agreed on certain rules that everyone must follow, so that ideas are always communicated clearly. To understand the need for a set of rules, try the following example.

Example 1

Evaluate the following expression. $3+4\times2$

Solution

One reasonable way to interpret this is to read it the way you would read a book, from left to right. Using this reasoning, we would first add, then multiply, and get $3+4\times2=7\times2=14$. But what if you thought to yourself, "multiplication is really just adding something a bunch of times. I know that $4\times2=2+2+2+2=8$. Why don't I multiply first, then add everything up?" Then the answer would be $3+4\times2=3+8=11$. Both answers are different! This shows that we must have some rules for the order in which we do things in math. In this lesson, we will learn the correct order to do operations.

One rule mathematicians decided was that multiplication comes before addition. (That means 11 is the correct answer in Example 1.)

But what about all the other operations, like division and subtraction?

Mathematicians have agreed on the following order of operations:

Algorithm

Order of Operations

1. Simplify Expressions in *Parentheses.*
2. Evaluate *Exponents*.
3. Perform all *Multiplication* and/or *Division* working from left to right
4. Perform all *Addition* and/or *Subtraction* working from left to right

One way we can remember the right order to calculate expressions is from one of the following phrases:

"Please Excuse My Dear Aunt Sally," or, if you prefer, "PEMDAS."

Please	**P**arenthesis
Excuse	**E**xponents
My **D**ear	**M**ultiplication or **D**ivision
Aunt **S**ally	**A**ddition or **S**ubtraction

Example 2

Simplify $4\times(6+2)$

Solution

First, we see that this expression involves multiplication and addition, and also contains a set of parentheses. The correct order of operations we should use is:

$4\times(6+2)=4\times8$ (Addition inside Parentheses)

$=32$ (Multiplication)

Example 3

Simplify $9+5\times3$

Solution

$9+5\times3=9+15$ (Multiplication)

$=24$ (Addition)

Example 4

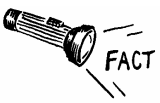
Simplify $5(3^2-6)$

Solution

$5(3^2-6)=5(9-6)$ (Exponent inside Parentheses)

$=5(3)$ (Subtraction inside Parentheses)

=15 (Multiplication)



Even though there is no "×" sign, a number next to parentheses means you should multiply.

Example 5

Simplify $4+3\times5-(6-2)^2\div2$

Solution

$4+3\times5-(6-2)^2\div2$ (Parentheses)

$=4+3\times5-4^2\div2$ (Exponents)

$=4+3\times5-16\div2$ (Multiplication and Division)

$=4+15-8$ (Addition and Subtraction)

$=11$

Simplify each expression.

1. $(4+5)^2$

2. $3^2+2\times7+3$

We now understand that parentheses are a way to group terms. They tell us to evaluate what is inside of them first. Parentheses can look different from what you are used to. Some common ways to write parentheses are with braces **{ }**, and brackets [ ]. Even though each set of parentheses looks different, they mean the same thing. We use different looking parentheses

sometimes, to make number phrases easier to read. For instance, $((2+3)\times(3-1))^2$ looks better when we write it as $\left[(2+3)\times(3-1)\right]^2$.

At times, parentheses are being used, but they are hiding. This happens whenever we see a fraction. For instance, the expression $\frac{7+1}{9-5}$ really means $(7+1)\div(9-5)$. We know how to simplify expressions in this form. Notice that a fraction is another way to show division. We will explore this in unit two.

Re-write the fraction using parentheses and the $\div$ sign.
Then simplify the expression.

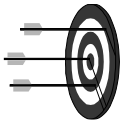
3. $\frac{4^2-4\times2}{3+1}$

4. $\frac{(12-6)^2-10\times3}{3\times2}$

Review

1. In your own words, explain why it is important to have a rule that tells us what the right order of operations is.

2. Write one thing you learned in this lesson, or one thing that you would like to ask your mentor.



Practice Problems

Unit 1 Lesson 7

Directions: Write your answers in your math journal. Label this exercise Unit 1 – Lesson 7, Set A and Set B.

Set A

1. Simplify the following expressions.

   (*Hint*: It might help you to first write what operations are being used, then to simplify using PEMDAS, or to first rewrite the expression in a form that is easier to work with.)

a. $(10+2)^2$ b. $5^2+4\times8\div2-12$ c. $\dfrac{7^2}{7}$

d. $\dfrac{3^3\times(3-2)^2}{2^3+1}$ e. $2[14+3(2)]$ f. $\dfrac{\left(\frac{4}{2}\right)^2}{2}$

Set B

1. Using math symbols and what you have learned about the order of operations, write an expression that means, "multiply three plus one by four, then divide by six." Do not solve the expression.

2. Daniel wants to write an expression for a number *x* that means, "after adding three to *x*, square it, then divide everything by two." Show Daniel how he would do this using math symbols and the correct order of operations.
   Hint: Treat *x* as if it were any other number; letters aren't that scary to work with!

1. $(4+5)^2=9^2=81$

2. $3^2+2\times7+3=9+2\times7+3=9+14+3=26$

3. $\dfrac{4^2-4\times2}{3+1}=(4^2-4\times2)\div(3+1)$
$=(16-4\times2)\div(4)=(16-8)\div4=8\div4=2$

4. $\dfrac{(12-6)^2-10\times3}{3\times2}$
$=[(12-6)^2-10\times3]\div6$
$=[6^2-10\times3]\div6$
$=[36-10\times3]\div6$
$=[36-30]\div6$
$=6\div6=1$

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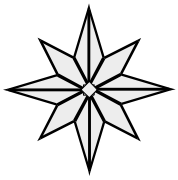

End of Lesson 7