Lesson 4

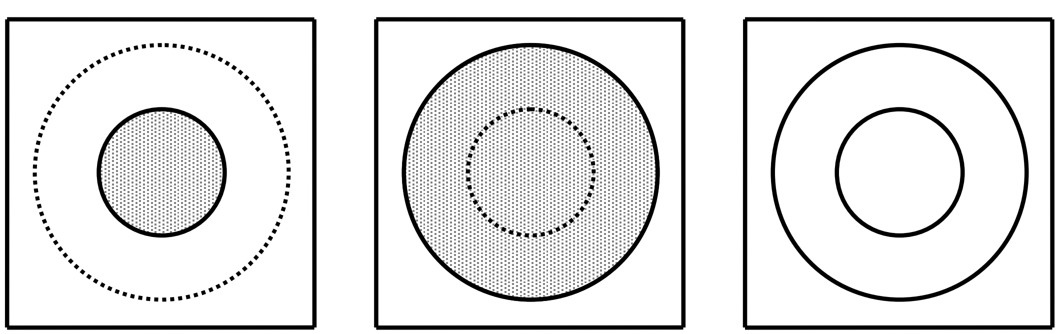
Conditional Statements

Objectives

- ✓ Analyze and write conditional statements
- ✓ Relate truth tables and conditional statements
- ✓ Write conditional statements symbolically

Think Back
*A **conditional statement** is a statement that can be written in if-then form. It is written symbolically $p \Rightarrow q$ and read "p implies q" or "if p then q."*

A Venn diagram used to represent a conditional statement often shows *p* inside *q*. Anything that applies to *p* applies to *q*. Anything in circle *p* is in circle *q*. If *p*, then *q*.



p ***q*** $\boldsymbol{p \Rightarrow q}$

Think Back

*The **hypothesis** (p) is the "if" part of a conditional statement.*

Think Back

*The **conclusion** (q) is the "then" part of a conditional statement.*

Sometimes the implication is obvious and sometimes it is not as obvious.

Example 1

These sentences can be written as conditional statements. Write each sentence in if-then form and find *p* and *q*.

a. If a quadrilateral is a square, it is a rectangle.

b. All even numbers are divisible by two.

c. Isosceles triangles have equal base angles.

Solution

a. If a quadrilateral is a square, then the quadrilateral is a rectangle.

p: A quadrilateral is a square.

q: The quadrilateral is a rectangle.

(You may also say: *If a quadrilateral is a square, then it is a rectangle*; however, saying *the quadrilateral* is clearer than saying *it.*)

b. If a number is even, then the number is divisible by two.

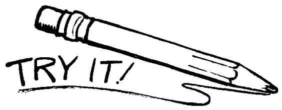
p: A number is even.

q: The number is divisible by two.

c. If a triangle is isosceles, then the triangle's base angles are equal.

p: A triangle is isosceles.

q: The triangle's base angles are equal.



1. These sentences can be written as conditional statements. Write each sentence in if-then form and find p and q.

a. If a line is perpendicular to one of two parallel lines, it is perpendicular to the other.

b. An equilateral triangle is equiangular.

Example 2

Suppose that I make the following statement. "If it rains, I will go to the movie."
Consider these events that follow my statement and determine if I kept my word.

It rained and I went to the movie.

Solution

I kept my word.

It rained and I did not go to the movie.

Solution

I did not keep my word.

Suppose it does not rain. Does it matter if I go to the movies or not?

Solution

I made no promise as to what I would do if it didn't rain, only what I would do if it does rain. Therefore whether or not I go to the movie is inconsequential.

The only time that the conditional statement "If it rains, I will go to the movie," is false is if the hypothesis is true (It rains) and the conclusion is false (I don't go to the movie).

Here is the truth table for a conditional statement.

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

The only time a conditional statement is false is if the hypothesis is true and the conclusion is false.

TRY IT!

2. Are the following conditional statements true or false?

a. If five is less than six, five is less than 10.

b. If seven is more than ten, seven is more than six.

c. If eleven is odd, eleven is divisible by two.

d. If eight is even, eight is a multiple of six.

Example 3

Complete this truth table for $p \Rightarrow \sim q$.

p	q	$\sim q$	$p \Rightarrow \sim q$
T	T		
T	F		
F	T		
F	F		

Solution

First fill in the ~*q* column and then ignore the second column (*q*) and use the first (*p*) and third (~*q*) columns to determine the truth-values in the last column.

p	q	$\sim q$	$p \Rightarrow \sim q$
T	T	F	F
T	F	T	T
F	T	F	T
F	F	T	T

3. Complete the truth-table for $\sim p \Rightarrow q$.

p	$\sim p$	q	$\sim p \Rightarrow q$
T		T	
T		F	
F		T	
F		F	

4. Why do you think it was a good idea to put the ~*p* column after *p* instead of after *q*?

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Example 4

Consider these two statements.

p: Point P is a point on the perpendicular bisector of a segment.

q: Point P is equidistant from the ends of the segment.

Write the following conditional statements symbolically.

If P is a point on the perpendicular bisector of a segment, then P is not equidistant from the ends of the segment.

Solution

$p \Rightarrow \sim q$

If P is equidistant from the ends of the segment, then P is a point on the perpendicular bisector of the segment.

Solution

$q \Rightarrow p$

Example 5

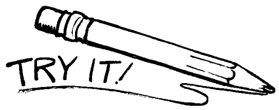
Using the same *p* and *q* from the last example, write $q \Rightarrow \sim p$ in words.

Solution

If P is equidistant from the ends of a segment, then P is not a point on the perpendicular bisector of the segment.

Think Back

Don't forget that statements may be true or they may be false.



5. Consider these two statements.

p: P is a point on the bisector of an angle

q: P is equidistant from the sides of the angle

Write each of these conditional statements symbolically.

a. If a point, P, is equidistant from the sides of an angle, then P is a point on the bisector of the angle.

b. If P is not on the bisector of an angle, then it is not equidistant from the sides of the angle.

Example 6

If p is true and $\sim q$ is true, what is the truth-value of $p \Rightarrow q$?

Solution

Since $\sim q$ is true, q is false. The conditional statement is false since $\begin{matrix} p \Rightarrow q \\ \text{T} \Rightarrow \text{F} \end{matrix}$ is false.

If p is true and $\sim q$ is true, what is the truth-value of $q \Rightarrow \sim p$?

Solution

Since q is false, the conditional statement is true whether $\sim p$ is true or false.

6. If p is false and ~q is true what is the truth-value of $q \Rightarrow \sim p$?

7. If p is false and ~q is true, what is the truth-value of $\sim p \Rightarrow q$?

8. If q is true and $\sim p \Rightarrow \sim q$ is true, what is the truth-value of p?

Review

1. Highlight:

- **a.** the definitions of a conditional statement, hypothesis, and conclusion.
- **b.** the truth table for a conditional statement.

2. Write down any questions you would like to discuss with your mentor.

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Practice Problems

Unit 3 Lesson 4

Directions: Write your answers in your math journal. Label this exercise Unit 3 – Lesson 4.

Connections/Modeling

1. Write each of these statements in if-then form and find the hypothesis, p, and the conclusion, q.

- **a.** If I fall down, I will need a bandage.
- **b.** Every living animal needs food.
- **c.** I will go to the party if I finish my homework.
- **d.** If it snows, my shovel will come in handy.

2. Tell whether each of these conditional statements is true or false.

a. If a square has a right angle, a square has unequal sides.

b. If a square has unequal sides, a square has a right angle.

c. If nine is a perfect cube, nine is less than 8.

d. If congruent figures have the same shape, they are regular polygons.

e. If similar figures have the same shape, they have proportional sides.

3. Complete this table for $\sim p \Rightarrow \sim q$.

p	$\sim p$	q	$\sim q$	$\sim p \Rightarrow \sim q$
T		T		
T		F		
F		T		
F		F		

4. Consider the two statements below.

p: Nine is a perfect square.

q: Ten is not a perfect square.

Write each of the following symbolically.

a. Nine is a perfect square and ten is a perfect square.

b. Ten is not a perfect square, if nine is not a perfect square.

c. Nine is a perfect square or ten is a perfect square.

d. If ten is not a perfect square, nine is a perfect square.

e. If nine is a perfect square, then ten is a perfect square.

5. If $\sim p$ is true and q is false, find the truth-value of each of these.

a. $p \vee q$

b. $q \Rightarrow p$

c. $\sim p \wedge \sim q$

d. $\sim p \Rightarrow \sim q$

e. $p \Rightarrow \sim q$

6. Complete the following truth table if 1 is true and 0 is false.

p	q	$p \wedge q$	$p \vee q$	$p \Rightarrow q$
1	1			
1	0			
0	1			
0	0			

7. Write as many statements as you can that use one *p* and one *q*, ***at least one*** ~, and ***exactly one*** of these symbols $\wedge$, $\vee$, $\Rightarrow$. You may not use parentheses. Write *p* first.

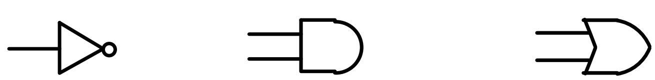
8. First fill in the "not" columns. Then label the column headings by choosing from your list of statements in Question 7 that result in the given truth-values.

p	$\sim p$	q	$\sim q$						
T		T		F	T	F	F	F	T
T		F		F	T	T	T	F	T
F		T		T	T	T	F	F	F
F		F		F	F	T	F	T	T

Explorations

1. Find a statement from your list in Connections/Modeling problem 7 that has the same truth-value outcomes as $p \Rightarrow q$.

2. Draw a schematic diagram using the three basic gates



to represent $p \Rightarrow q$.

3. Test your diagram in the last problem by making:

a. *p* true and *q* true

b. *p* true and *q* false

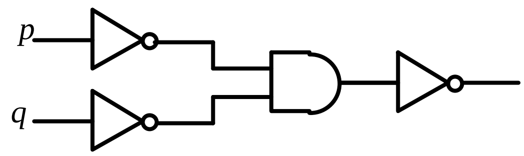
c. *p* false and *q* true

d. *p* false and *q* false

and finding the truth-values along the path each time, including the final output of the device.

Justifications/Proofs

1. What single gate does the following schematic diagram represent? Justify your answer.



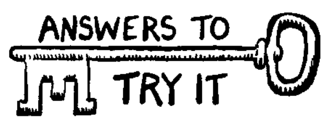
2. If p is true and $p \Rightarrow \sim q$ is false, what is the truth-value of q? Justify your answer.

3. If q is true and $\sim p \Rightarrow \sim q$ is false, what is the truth-value of p? Justify your answer.

4. Prove that $p \Rightarrow p$ is always true.

5. If $p \Rightarrow q$ is true and p and q have opposite truth-values, what is the truth-value of p? Justify your answer.

6. If $p \Rightarrow q$ is true and q is true, what is the truth-value of p? Justify your answer.



1. **a.** If a line is perpendicular to one of two parallel lines, then that line is perpendicular to the other.

p: A line is perpendicular to one of two parallel lines.

q: The line is perpendicular to the other parallel line.

b. If a triangle is equilateral, then the triangle is equiangular.

p: A triangle is equilateral.

q: The triangle is equiangular.

2. **a.** $\mathrm{T} \Rightarrow \mathrm{T}$ is true.

b. $\mathrm{F} \Rightarrow \mathrm{T}$ is true. Note that the conditional statement does not actually guarantee that seven is more than ten but only what happens if it is true.

c. $\mathrm{T} \Rightarrow \mathrm{F}$ is false. Since eleven is odd, then in order for the conditional statement to be true, eleven would have to be divisible by two, which it is not.

d. $\mathrm{T} \Rightarrow \mathrm{F}$ is false.

3.

p	**~*p***	***q***	$\sim p \Rightarrow q$
T	**F**	**T**	T
T	**F**	**F**	T
F	**T**	**T**	T
F	**T**	**F**	F

4. It is probably easier to compare p and ~p if they are next to each other. It is certainly easier to determine the truth-value of an implication, $\Rightarrow$, when the statements being compared are in the proper order and order does matter as the arrow would seem to indicate. You may have other ideas as well.

5. **a.** $q \Rightarrow p$ **b.** $\sim p \Rightarrow \sim q$

6. Since $\sim q$ is true, q is false. Therefore the conditional statement is true no matter what the truth-value of p or ~p.

7. Since p is false, then ~p is true. Since ~q is true, then q is false. $\text{T} \Rightarrow \text{F}$ is false so $\sim p \Rightarrow q$ is false.

8. If q is true, then ~q is false. $\text{T} \Rightarrow \text{F}$ is false, but $\text{F} \Rightarrow \text{F}$ is true. So if ~q is false, then ~p must be false for $\sim p \Rightarrow \sim q$ to be true. This means that p is true.

End of Lesson 4