

**Materials Needed:**  
 compass  
 straight edge

# Lesson 7

## Area of Regular Polygons

### Objectives

- ✓ Define apothem
- ✓ Discover, prove, and use the formula for finding the area of a regular polygon



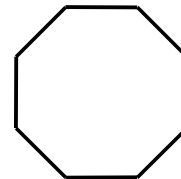
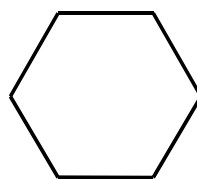
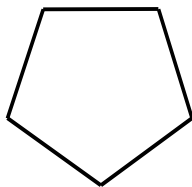
**Think Back**

*A regular polygon is a convex polygon that is both equilateral and equiangular.*

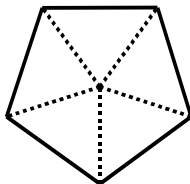
### EXPLORATION



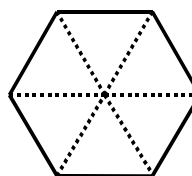
Into how many congruent triangles can each of the following regular polygons be divided?



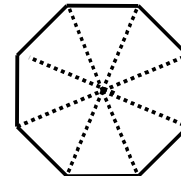
**Solution**



5 triangles



6 triangles




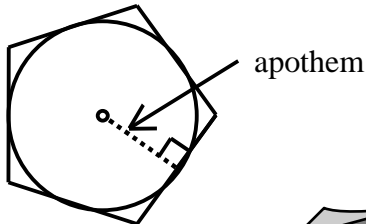
8 triangles

Into how many congruent triangles can any regular polygon with  $n$  sides be divided?

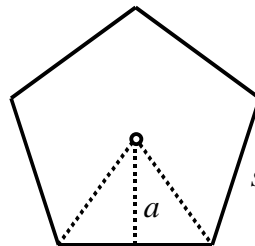
**Solution**

There will be  $n$  congruent triangles.

 An **apothem** is the radius of the inscribed circle of a regular polygon.



1. Write a formula for the area of one of the congruent triangles of a regular polygon in terms of its apothem and length of one of its sides.



2. Make a conjecture about finding the area of a regular  $n$ -sided polygon in terms of its apothem,  $a$ , and the length of one of its sides,  $s$ .

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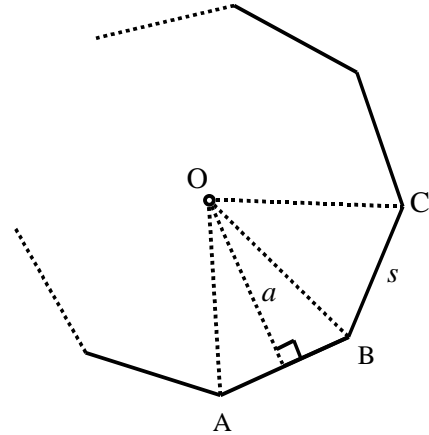
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**Proposition 28: Theorem 4.4**

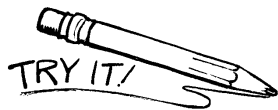
The area of a regular polygon is equal to half the product of its apothem and its perimeter.

Given: Regular polygon  $ABC\cdots$  with  $n$  sides,  
length of each side  $s$ , perimeter  $p$ , apothem  $a$ ,  
and center  $O$ .

To Prove: Area of  $ABC\cdots = \frac{1}{2}ap$



Statements	Reasons
1. Regular polygon $ABC\cdots$ with $n$ sides, length of each side $s$ , perimeter $p$ , apothem $a$ , and center $O$	1. Given
2. Draw $\overline{OA}$ , $\overline{OB}$ , $\overline{OC}$ , $\cdots$ forming $n$ congruent triangles.	2. The polygon has $n$ sides (St. 1).
3. The area of each triangle is $\frac{1}{2}sa$ .	3. The area of a triangle is equal to half the product of its base and its altitude (St. 1).
4. The area of $ABC\cdots =$ the area of $n$ triangles $= \frac{1}{2}nsa$ .	4. The whole of a quantity is equal to the sum of its parts.
5. $p = ns$	5. Definition of perimeter
6. The area of $ABC\cdots = \frac{1}{2}pa$	6. A quantity may be substituted for its equal in any expression (St. 4 & 5).
7. The area of $ABC\cdots = \frac{1}{2}ap$	7. Commutative Property



3. Explain in your own words why the sketch in the previous theorem was incompletely drawn.

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**Example 1**

How many sides does a regular polygon have if it has an area of 750 square units, an apothem of 15 units, and the length of each side is 12.5 units.

$$A = \frac{1}{2}ap$$

**Solution**

$$750 = \frac{1}{2}(15)p$$

The perimeter of the polygon is  $100 = p$

$$p = ns$$

The number of sides are  $100 = 12.5n$

$$8 = n$$

The polygon is an octagon.



4. Find the length of a side of a regular hexagon with an apothem of 10 inches and an area of 348 square inches.

 **Review**

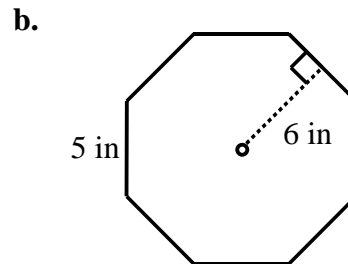
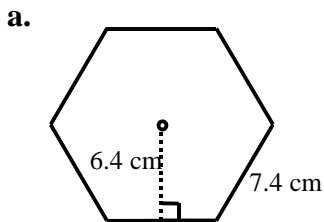
1. Highlight Theorem 4.4 and the definition of apothem.
  2. Write down any questions you would like to discuss with your mentor.
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 **Practice Problems**  
**Unit 4 Lesson 7**

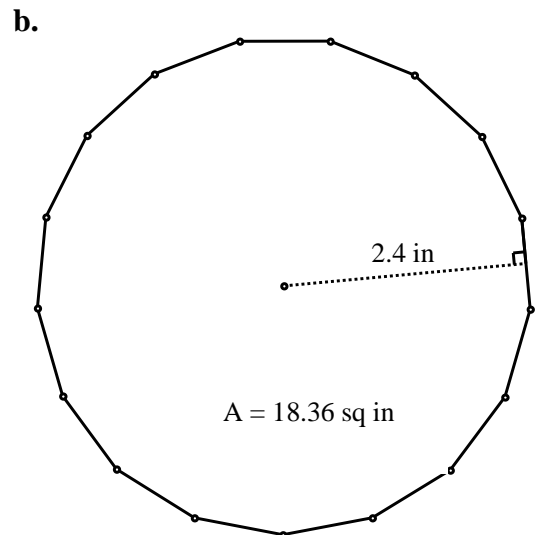
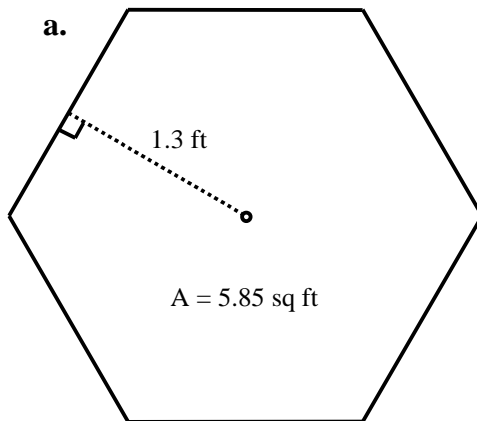
Directions: Write your answers in your math journal. Label this exercise Unit 4 – Lesson 7.

***Connections/Modeling***

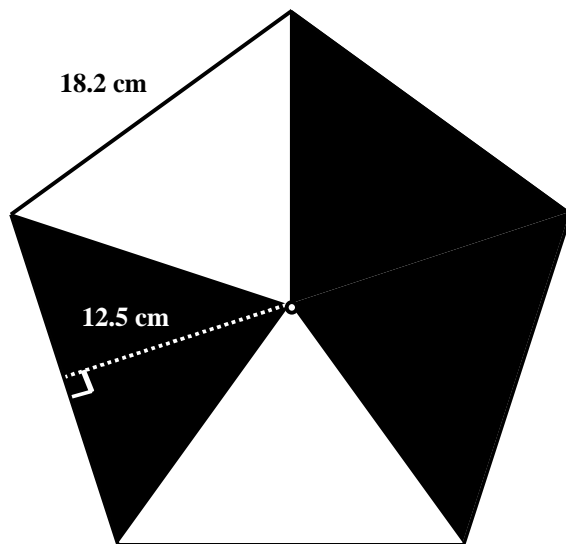
1. Find the area of each regular polygon.



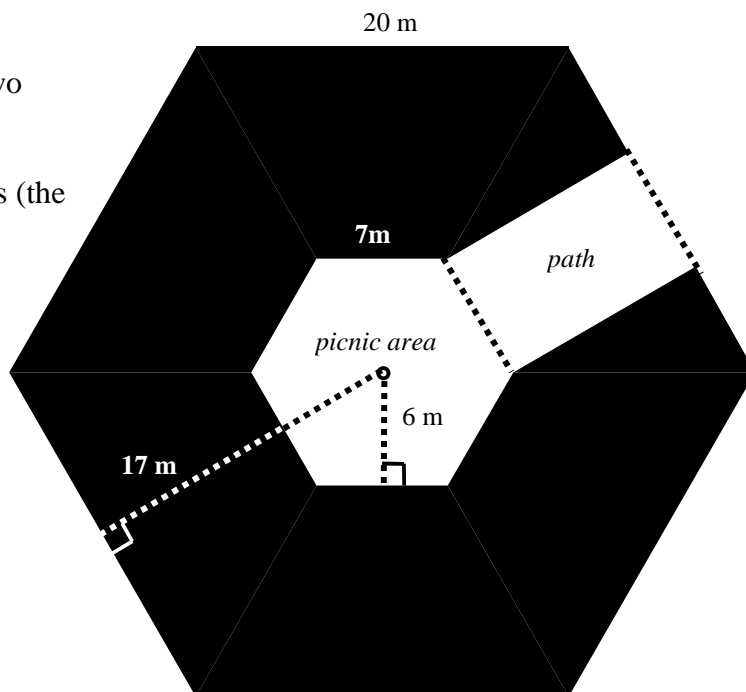
2. Find the perimeter and the length of each side of these regular polygons.



3. Find the number of sides of a regular polygon with:
  - a. Area 12.48 square yards, apothem 2 yards, and length of each side 0.78 yards.
  - b. Area 897.6 square units, apothem 1.7 units, and length of each side 0.88 units.
4. Find the area of the shaded region inside this regular polygon.

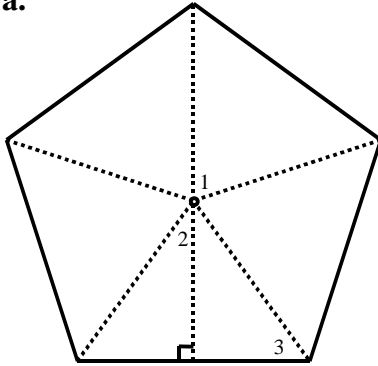


5. The park area at right is made up of two regular concentric polygons and a rectangle. Find the area of the gardens (the shaded region).

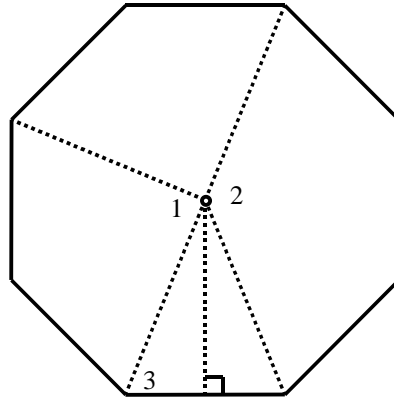


6. Find the measure of each of the marked angles in the regular polygons.

a.



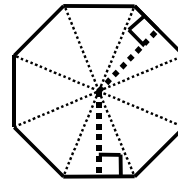
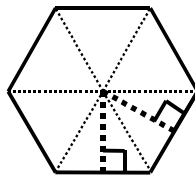
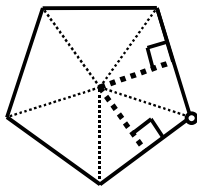
b.



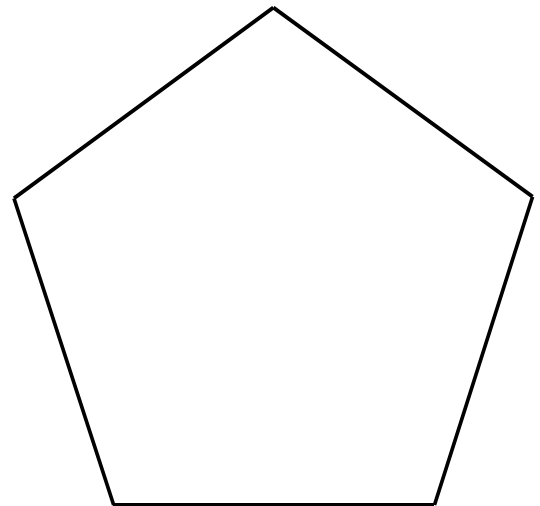
**Explorations**

1. To inscribe a circle in a regular polygon and circumscribe a circle about a regular polygon consider the following.

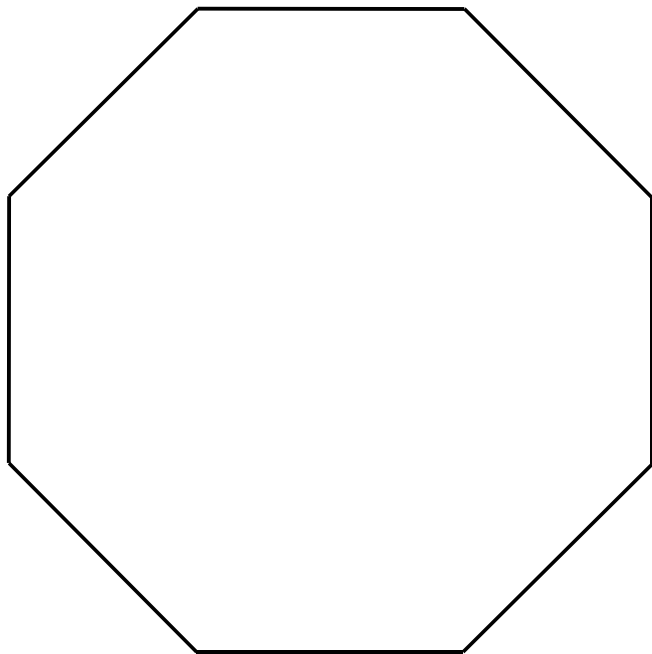
- a. What two things must you know in order to draw a circle?
- b. Where is the center of a regular polygon? Study the figure before you answer.



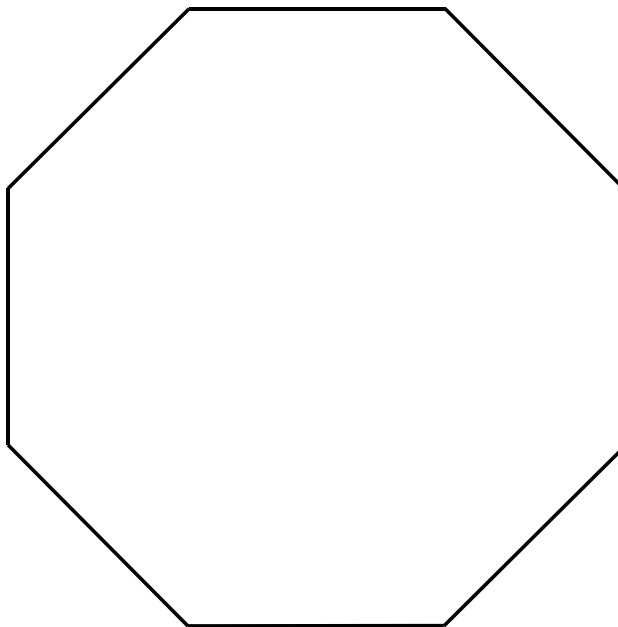
- c. Given a regular polygon, how could you find (by construction) its center?
- d. Find the center of the regular polygon.



2. To circumscribe a circle about a regular polygon you must find the center (Exploration 1) of the circle and its radius.
- a. How could you find the radius of the circumscribed circle?
  - b. Circumscribe a circle about the regular polygon.



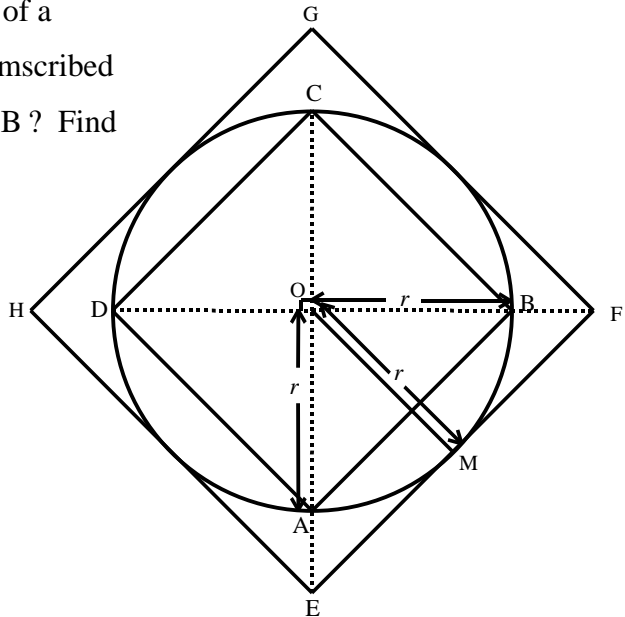
3. To inscribe a circle in a regular polygon you must find its center (Exploration 1) and its radius.
- a. How could you find the radius of the inscribed circle?
  - b. Inscribe a circle in this regular polygon.





*Justifications/Proofs*

1. Since the center of a regular polygon is at the intersection of the perpendicular bisector of all the sides of the polygon, explain why is it only necessary to find the perpendicular bisector of any two of the sides instead of all of them in order to find the center?
2. Write a two-column proof showing that the apothem of a regular polygon bisects any of the sides to which it is drawn.
3. Write a paragraph proving that if all the diagonals of a regular pentagon are drawn from any vertex of the pentagon, they will divide the angle into three equal angles.
4. Write a paragraph proving that if all the diagonals of a regular hexagon are drawn from any vertex of the hexagon, they will divide the angle into four equal angles.
5. Prove that the area of an inscribed square of a circle is equal to half the area of the circumscribed circle. Hint: What kind of angle is  $\angle AOB$ ? Find  $m\overline{AB}$  and  $m\overline{EF}$  in terms of the radius  $r$ .





1.  $A = \frac{1}{2}sa$
2. Conjecture: The area of a regular  $n$ -sided polygon with apothem  $a$  and length of side  $s$  is  $n(\frac{1}{2}sa)$ .
3. The sketch is a sketch of a general polygon. The number of its sides is not specific. If the polygon were to be finished, it would have a specific number of sides.
4. First find the perimeter.  $A = \frac{1}{2}ap$   
 $348 \text{ sq in} = \frac{1}{2}(10 \text{ in})p$   
 $69.6 \text{ in} = p$

$$ns = p$$

Then find the length of a side.  $6s = 69.9 \text{ in}$   
 $s = 11.6 \text{ in}$



**End of Lesson 7**