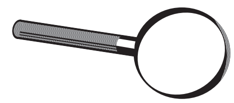
Lesson 8
Properties of Logarithms

Objectives



In this lesson you will:

- ✓ Discover the properties of logarithms through investigations.
- ✓ Use the product, quotient, and power properties of logarithms.
- ✓ Expand or condense a logarithmic expression.
- ✓ Evaluate a logarithmic expression using the change-of-base formula.

Investigating Properties of Logarithms

In Lessons 6 and 7 of this unit, you learned to evaluate and graph logarithms. In this lesson you will discover some special properties of logarithms that will make it easier to use logarithms in more complicated expressions and in solving problems.

Use your TI-83 Plus calculator (or any calculator that has a log key) to find the missing values in the table below. Round values to the nearest thousandth.

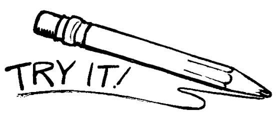
m	***n***	**log *m***	**log *n***	***mn***	**log *mn***
40	30	1.602	1.477	40(30) = 1200	3.079
10	25	1.000	1.398	10(25) = 250	?
20	20	1.301	?	20(20) = 400	2.602
49	45	?	1.653	49(45) = 2205	3.343
100	10	2	?	?	3

In the table on the previous page, you should have replaced the question marks with the numbers 2.398, 1.301, 1.690, 1, and 100(10) = 1000. This table demonstrates what is known as the **product property**. The product property states:

$$\log_b m + \log_b n = \log_b mn,$$

where m, n, and b are positive integers greater than 0 and $b \neq 1$.

In the table, the base of each logarithm is 10. Remember that it is not necessary to write the base when it is 10.



1. State what the product property means in your own words.

2. If log 240 = 2.38 and log 1000 = 3, find log 240,000 without using a calculator.

3. If $\log_2 8 = 3$ and $\log_2 4 = 2$, what is $\log_2 32$?

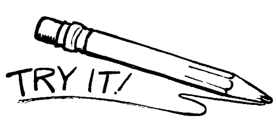
Use your calculator to find the missing values in the table below. Round values to the nearest thousandth.

m	n	**log m**	**log n**	$\frac{m}{n}$	**log** $\frac{m}{n}$
40	20	1.602	1.301	40/20 = 2	.301
45	9	1.653	.954	45/9 = 5	?
20	20	1.301	?	20/20 = 1	?
90	30	?	1.477	90/30 = 3	.477
1000	10	3	1	?	?

In the table above, you should have replaced the question marks with the numbers .699, 1.301, 0, 1.954, 1000/10 = 100, and 2. This table demonstrates what is known as the **quotient property**. The quotient property states:

$$\log_b m - \log_b n = \log_b \frac{m}{n},$$

where m, n, and b are positive integers greater than 0 and $b \neq 1$.

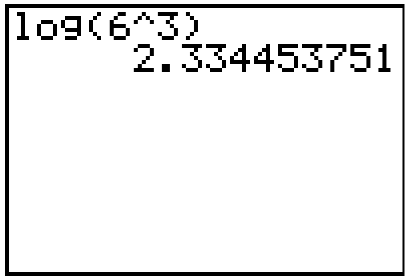


4. State what the quotient property means in your own words.

5. If log 317 = 2.501 and log 200 = 2.301, find $\log \frac{317}{200}$ without using a calculator.

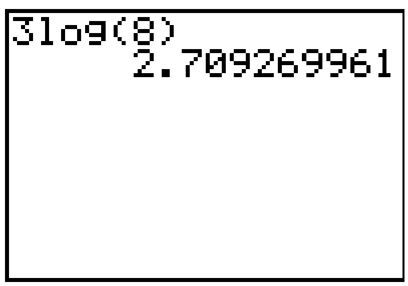
To evaluate an expression like log (6^3) on the TI-83 Plus calculator, do the following:

Press the log key, then 6, followed by the ^ key, and 3. Then close the parentheses and press ENTER .Your screen should look like this:

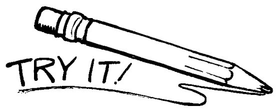


You can see from the screen that log (6^3) is approximately 2.334.

To evaluate an expression like 3 log 8 (read "three times the log of 8") on the calculator, press 3, then the log key, then 8. Close the parentheses and press ENTER .Your screen should look like this:



You can see from the screen that 3 log 8 is approximately 2.709.



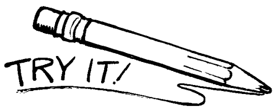
Evaluate each logarithmic expression with a calculator and write its value in the space following the expression. Then match each expression in the first column with its equivalent expression in the second column.

6. $\log(2^3)=$ _______	A. 4 log 5
7. $\log(4^5)=$ _______	B. 3 log 2
8. $\log(3^2)=$ _______	C. 4 log 2
9. $\log(5^4)=$ _______	D. 2 log 3
10. $\log(2^4)=$ _______	E. 2 log 5
11. $\log(5^2)=$ _______	F. 5 log 4

12. Study each pair of equivalent expressions above. Do you notice anything special about them? Using what you learned in the exercise above, can you write an equivalent expression for $\log(7^5)$?

While doing the Try It! exercises above, you probably discovered the following property, known as the **power property:**

$$\log_b n^p = p \cdot \log_b n$$



$\text{Log}_4 6 \approx 1.2925$ and $\log_4 20 \approx 2.16097$. **Use one of the properties of logarithms to evaluate the following expressions.**

13. $\log_4 (6^5)$ **14.** $\log_4 (6 \cdot 20)$ **15.** $\log_4 \dfrac{10}{3}$

Expanding or Condensing a Logarithmic Expression

Sometimes it may be helpful to expand or condense a logarithmic expression. To **expand** a logarithmic expression means to write it as a longer equivalent expression. To **condense** a logarithmic expression means to write it as a shorter equivalent expression. You can use the properties that you just learned to expand or condense logarithmic expressions. In the following examples, you may assume that x and y are positive.

Example 1

Expand the expression $\log_3 \dfrac{10x}{y}$.

Solution

$\log_3 \dfrac{10x}{y}$	Write the original expression.
$\log_3 10x - \log_3 y$	Use the quotient property.
$\log_3 10 + \log_3 x - \log_3 y$	Use the product property.

Example 2

Expand the expression $\log 8(x^3)$.

Solution

$\log 8(x^3)$	Write the original expression.
$\log 8 + \log x^3$	Use the product property.
$\log 8 + 3 \log x$	Use the power property.

Example 3

Condense the expression ln 12 – ln 3.

Solution

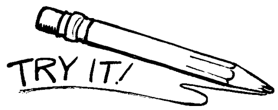
$\ln 12 - \ln 3$	Write the original expression.
$\ln \frac{12}{3}$	Use the quotient property.
$\ln 4$	Simplify.

Example 4

Condense the expression $3 \log x + \log 9$.

Solution

$3 \log x + \log 9$	Write the original expression.
$\log x^3 + \log 9$	Use the power property.
$\log 9(x^3)$	Use the product property.



16. Expand the expression $\log x^4 y^3$.

17. Condense the expression $3 \ln x - 4 \ln 2$.

Using the Change-of-Base Formula

You have learned how to evaluate common logarithms (base 10) and natural logarithms (base *e*) on the calculator. You may have wondered how to evaluate logarithms that have a base other than 10 or *e*. We can rewrite a logarithm with any base in terms of common or natural logarithms. To do this we must use the **change-of-base formula**.

$$\log_a n = \frac{\log_b n}{\log_b a}$$

In the change-of-base formula, *a*, *b*, and *n* are positive. Also, neither *a* nor *b* is 1. When we rewrite the expression using common logarithms, *b* will be 10. When we rewrite the expression using natural logarithms, *b* will be *e*.

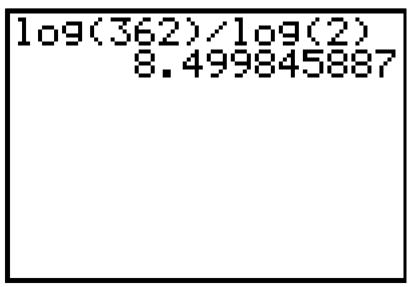
Example 5

Evaluate the expression $\log_2 362$ using the change-of-base formula and a calculator. Use **(a)** common logarithms, then **(b)** natural logarithms. Round to the nearest thousandth.

Solution

(a) $\log_2 362$	Write the original expression.
$\dfrac{\log 362}{\log 2}$	Use the change-of-base formula.

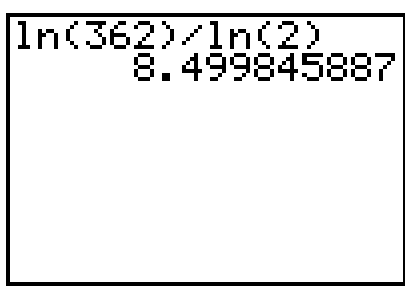
Now, on your calculator press log , 362,) , ÷ , log , 2,) , and ENTER . If you do not close the parentheses, you could get an incorrect answer. Your screen should look like this:



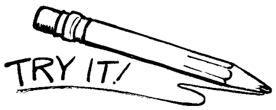
Therefore, $\log_2 362$ is approximately 8.500.

(b) $\log_2 362$	Write the original expression.
$\dfrac{\ln 362}{\ln 2}$	Use the change-of-base formula.

Now, on your calculator press ln , 362,) , ÷ , ln , 2,) , and ENTER . Your screen should look like this:



Again, $\log_2 362$ is approximately 8.500.

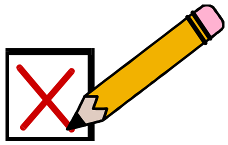


Use the change-of-base formula and a calculator to evaluate each expression. Round to the nearest thousandth.

18. $\log_9 6$

19. $\log_{17} 4913$

20. $\log_4 128$



Review

1. Locate and highlight the vocabulary words and their meaning in this lesson.
2. State what the **power property** says in your own words.
3. What does it mean to expand or condense a **logarithmic expression**?
4. In your own words, explain how to use the **change-of-base formula**.
5. Write one new thing that you learned from this lesson or one question that you would like to ask your mentor.

Practice Problems
Unit 1 Lesson 8

Directions: Write your answers in your math notebook. Label this exercise Unit 1 –Lesson 8, Set A, Set B, Set C and Set D.

Set A

$\text{Log}_5 10 \approx 1.431$, $\log_5 2 \approx 0.431$ and $\log_5 4 \approx 0.861$. Use one of the properties of logarithms to evaluate the following expressions.

1. $\log_5 2^3$ **2.** $\log_5 5$ **3.** $\log_5 40$

4. $\log_5 10^2$ **5.** $\log_5 8$ **6.** $\log_5 \frac{5}{2}$

Set B

Expand the following logarithmic expressions.

1. $\ln 5x$ **2.** $\log \frac{x}{y}$ **3.** $\log_6 x^3$

4. $\log \frac{4}{5y}$ **5.** $\ln xy^5$ **6.** $\log \frac{6x}{y^7}$

Set C

Condense the following logarithmic expressions.

1. $\ln x + \ln y$ **2.** $3 \log_2 5 - \log_2 x$ **3.** $\log 15 - 2 \log 3$

4. $\log_2 x + \log_3 x$ **5.** $\ln 3 + \ln 6 + \ln x$ **6.** $4 \log 2 - 2 \log 4$

Set D

Use the change-of-base formula and a calculator to evaluate each expression. Round to the nearest thousandth.

1. $\log_3 1000$ **2.** $\log_8 32{,}768$ **3.** $\log_5 35$

4. $\log_6 200$ **5.** $\log_2 2048$ **6.** $\log_7 153$

ANSWERS TO TRY IT

1. If two logarithmic expressions have the same base, then the logarithm of one number plus the logarithmic of a second number will be equal to the logarithm of the first number times the second number.

2. 2.38 + 3 = 5.38

3. 3 + 2 = 5

4. If two logarithmic expressions have the same base, then the logarithm of one number minus the logarithm of a second number will be equal to the logarithm of the first number divided by the second number.

5. 2.501 – 2.301 = .2

6. B **7.** F **8.** D **9.** A. **10.** C **11.** E

12. $\log 7^5 = 5 \log 7$

13. $5 \log_4 6 = 5(1.2925) = 6.4625$

14. $\log_4 6 + \log_4 20 = 1.2925 + 2.16097 = 3.45347$

15. $\log_4 \frac{10}{3} = \log_4 \frac{20}{6} = \log_4 20 - \log_4 6 = 2.16097 - 1.2925 = .86847$

16. $\log x^4 + \log y^3 = 4 \log x + 3 \log y$

17. $\ln x^3 - \ln 2^4 = \ln x^3 - \ln 16 = \ln \frac{x^3}{16}$

18. 0.815 **19.** 3.000 **20.** 3.500

End of Lesson 8