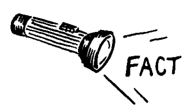
# Lesson 4

## Solving Quadratic Equations – By Factoring

### Objectives

learn

- ✓ the zero-product property
- ✓ to solve equations by factoring




**Zero-Product Property**

*If the product of two numbers is zero, at least one of the numbers is zero. If $ab = 0$, then either $a = 0$, $b = 0$, or they are both zero.*

***Example 1***

Find all the values of *x* that make this equation true. $(x-3)(x+5)=0$

***Solution***

$(x-3)=0$ or $(x+5)=0$ by the zero-product property.

$x-3=0 \qquad\qquad x+5=0$

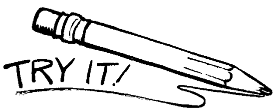
$x=3 \qquad\qquad x=-5$

The solution set is {3,–5}.

Check:

If $x=3$, then $(x-3)(x+5)=(3-3)(3+5)=0(8)=0$

If $x=-5$, then $(x-3)(x+5)=(-5-3)(-5+5)=-8(0)=0$




Find all the values of the variables that make these equations true.

Check your answers.

1. $(y+4)(y-2)=0$

2. $(2x-3)(x-1)=0$

3. $(5r+3)(4r+7)=0$

The zero-product property allows you to solve many quadratic equations by factoring.

***Example 2***

Use the zero-product property to help you find all the values of the variables that make each quadratic equation true.

$a^2-100=0$

***Solution***

This is the difference of two squares.

It may be factored as the sum and the difference of the two numbers that were squared.

$$a^2 - 100 = 0$$
$$(a-10)(a+10) = 0$$
$$a - 10 = 0 \quad \text{or} \quad a + 10 = 0$$
$$a = 10 \qquad\qquad a = -10$$

The solution set is $\{10,\ -10\}$

Check:

If $a = 10$, $a^2 - 100 = 10^2 - 100 = 100 - 100 = 0$.

If $a = -10$, than $a^2 - 100 = (-10)^2 - 100 = 100 - 100 = 0$.

$2x^2 - 13x - 7 = 0$

***Solution***

This is a trinomial with no common factors. Try factoring it as the product of two binomials.

$$2x^2 - 13x - 7 = 0$$
$$(2x+1)(x-7) = 0$$
$$2x + 1 = 0 \quad \text{or} \quad x - 7 = 0$$
$$2x = -1 \qquad\qquad x = 7$$
$$x = -\tfrac{1}{2}$$

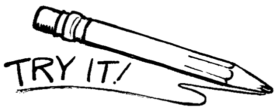
The solution set is $\{-\tfrac{1}{2},\ 7\}$

Check:

If $x = -\tfrac{1}{2}$, then $2x^2 - 13x - 7 = 2\left(-\tfrac{1}{2}\right)^2 - 13\left(-\tfrac{1}{2}\right) - 7 = \tfrac{1}{2} + 6\tfrac{1}{2} - 7 = 7 - 7 = 0$.

If $x = 7$, then $2x^2 - 13x - 7 = 2(7)^2 - 13(7) - 7 = 98 - 91 - 7 = 7 - 7 = 0$.

Review factoring techniques and then try the following problems.




Use the zero-product property to help you find all the values of the variables that make each quadratic equation true. Check your results.

4. $6x^2 - 17x - 14 = 0$

5. $2b^2 - 26b + 80 = 0$

6. $25x^2 - 16 = 0$

***Example 3***

A man throws a ball upward beside the edge of a cliff. It lands in the valley below. If the height $s$ of the ball $t$ seconds after it leaves the man's hand is $s = -16t^2 + 86t + 830$, how long is the ball in the air?

***Solution***

The ball will hit the ground when $s = 0$.

$$0 = -16t^2 + 86t + 830$$

$$0 = -2\left(8t^2 - 43t - 415\right)$$

$$0 = -2(8t - 83)(t + 5)$$

So $8t - 83 = 0$ or $t + 5 = 0$

$8t = 83$ or $t = -5$ sec.

$t \approx 10.4$ sec.

Since $-5$ seconds makes no sense in this problem, it is an extraneous root. The solution is approximately 10.4 seconds.

***Example 4***

The sum of the squares of two positive consecutive integers is 365. What are the numbers?

***Solution***

If $x$ is one of the integers, then the other integer is $x + 1$. Their squares are $x^2$ and $(x+1)^2$, respectively. The equation implied by the statement is $x^2 + (x+1)^2 = 365$.

$$x^2 + (x+1)^2 = 365$$

$$x^2 + x^2 + 2x + 1 = 365$$

$$2x^2 + 2x - 364 = 0$$

$$2\left(x^2 + x - 182\right) = 0$$

$$2(x - 13)(x + 14) = 0$$

So, either $x - 13 = 0$ or $x + 14 = 0$. This means that $x = 13$ or $x = -14$. Since the integers were positive, this means that $x = 13$ and $x + 1 = 14$.

The integers are 13 and 14.

Check: $13^2 + 14^2 = 169 + 196 = 365$.

TRY IT!

7. If a woman dives off a 64 foot cliff, the equation of her height $s$ in feet $t$ seconds after she jumps is $s = -16t^2 + 64$. How long will it take her to hit the water?

8. A rectangular field is 5 times as long as it is wide. If there are as many square yards in the area as there are yards in the perimeter, what are the dimensions of the rectangle?

## Review

1. Highlight the zero-product property.
2. Write about one new thing that you learned in this lesson or write a question that you would like to discuss with your mentor.

# Practice Problems

**Unit 3 Lesson 4**

Directions: Write your answers in your math journal. Label this exercise Unit 3-Lesson 4 Set A, Set B, and Set C.

***Set A***

Find all the values that make these equations true. Check your answers.

1. $(x-3)(x-5)=0$
2. $(4y-3)(y+6)=0$
3. $(6n+5)(2n+3)=0$
4. $(x+3)(x+3)=0$
5. $(x+8)(x-3)=0$
6. $(2x+5)(2x-1)=0$

***Set B***

Use the zero-product property to help you find all the values of the variables that make each quadratic equation true.

1. $4x^2-4x-15=0$
2. $3b^2-33b+84=0$
3. $49x^2-36=0$
4. $3x^2=2x$
5. $x^2=11x-30$
6. $6x^2-x-2=0$
7. $9x^2-6x+1=0$

***Set C***

1. A woman throws a ball upward beside the edge of a cliff. It lands in the valley below. If the height *s* of the ball *t* seconds after it leaves the woman's hand is $s=-16t^2-10t+516$, how long is the ball in the air?

2. The sum of the squares of two negative consecutive integers is 313. What are the numbers?

3. A rectangular field is 7 times as long as it is wide. If there are as many square yards in the area as there are yards in the perimeter, what are the dimensions of the field?

4. Does the zero-product property work for other numbers? For example, if the product of two numbers is 15, does one of them have to be 15? Justify your answer.

## ANSWERS TO TRY IT

1. $(y+4)(y-2)=0$

   $y+4=0$ or $y-2=0$

   $y=-4$ $\quad$ $y=2$

   The solution set is $\{-4, 2\}$

   Check:

   If $y=-4$, then $(y+4)(y-2)=0(-6)=0$

   If $y=2$, then $(y+4)(y-2)=6(0)=0$

2. $(2x-3)(x-1)=0$

   $2x-3=0$ or $x-1=0$

   $2x=3$ $\quad$ $x=1$

   $x=\frac{3}{2}$

   Check:

   If $x=\frac{3}{2}$, then $(2x-3)(x-1)=\left[\cancel{2}\left(\frac{3}{\cancel{2}}\right)-3\right]\left[\frac{3}{2}-1\right]=(3-3)\left(\frac{1}{2}\right)=0\left(\frac{1}{2}\right)=0$

   If $x=1$, then $(2x-3)(x-1)=(2\cdot1-3)(1-1)=(2-3)(0)=-1(0)=0$

3. $(5r+3)(4r+7)=0$

   $5r+3=0$ or $4r+7=0$

   $5r=-3$ $\quad$ $4r=-7$

   $r=-\frac{3}{5}$ $\quad$ $r=-\frac{7}{4}$

   Check:

   If $r=-\frac{3}{5}$, then

   $(5r+3)(4r+7)=\left[\cancel{5}\left(-\frac{3}{\cancel{5}}\right)+3\right]\left[4\left(-\frac{3}{5}\right)+7\right]=(-3+3)\left(-\frac{12}{5}+7\right)=0\left(-\frac{12}{5}+7\right)=0$

   If $r=-\frac{7}{4}$, then

   $(5r+3)(4r+7)=\left[5\left(-\frac{7}{4}\right)+3\right]\left[\cancel{4}\left(-\frac{7}{\cancel{4}}\right)+7\right]=\left(-\frac{35}{4}+3\right)(-7+7)=\left(-\frac{35}{4}+3\right)0=0$

4. $6x^2-17x-14=0$

   $(2x-7)(3x+2)=0$

   $2x-7=0$ or $3x+2=0$

   $2x=7$ $\quad$ $3x=-2$

   $x=\frac{7}{2}$ $\quad$ $x=-\frac{2}{3}$ $\quad$ The solution set is $\{\frac{7}{2}, -\frac{2}{3}\}$

   Check:

   If $x=\frac{7}{2}$, then $6x^2-17x-14=6\left(\frac{7}{2}\right)^2-17\left(\frac{7}{2}\right)-14=\frac{147}{2}-\frac{119}{2}-14=14-14=0$.

   If $x=-\frac{2}{3}$, then $6x^2-17x-14=6\left(-\frac{2}{3}\right)^2-17\left(-\frac{2}{3}\right)-14=\frac{8}{3}+\frac{34}{3}-14=14-14=0$

5. $2b^2 - 26b + 80 = 0$

$2\left(b^2 - 13b + 40\right) = 0$

$2\left(b - 8\right)\left(b - 5\right) = 0$

$b - 8 = 0$ or $b - 5 = 0$

$b = 8$ $b = 5$ The solution set is {8, 5}.

Check:

If $b = 8$, then $2b^2 - 26b + 80 = 2\left(8^2\right) - 26\left(8\right) + 80 = 128 - 208 + 80 = 0$.

If $b = 5$, then $2b^2 - 26b + 80 = 2\left(5^2\right) - 26\left(5\right) + 80 = 50 - 130 + 80 = 0$.

6. $25x^2 - 16 = 0$

$\left(5x + 4\right)\left(5x - 4\right) = 0$

$5x + 4 = 0$ or $5x - 4 = 0$

$x = -\frac{4}{5}$ $x = \frac{4}{5}$

Check:

If $x = -\frac{4}{5}$, then $25x^2 - 16 = 25\left(-\frac{4}{5}\right)^2 - 16 = 16 - 16 = 0$.

If $x = \frac{4}{5}$, then $25x^2 - 16 = 25\left(\frac{4}{5}\right)^2 - 16 = 16 - 16 = 0$.

7. She will hit the water when $s = 0$.

$0 = -16t^2 + 64$

$0 = \left(8 - 4t\right)\left(8 + 4t\right)$

$8 - 4t = 0$ or $8 + 4t = 0$

$8 = 4t$ $8 = -4t$

$2 = t$ $-2 = t$

The only solution that makes sense is $t = 2$ seconds.

This could also be solved using the techniques of the last lesson.

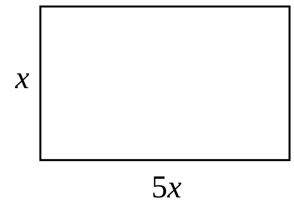
$0 = -16t^2 + 64$

$16t^2 = 64$

$t^2 = 4$

$t = \pm 2$

8. Make a sketch.




Area = Perimeter

$x(5x) = 2x + 2(5x)$

$5x^2 = 12x$

$5x^2 - 12x = 0$

$x(5x - 12) = 0$

$x = 0$ or $5x - 12 = 0$

$5x = 12$

$x = \frac{12}{5}$

Only the second answer is reasonable. The width is $2\frac{2}{5}$ yds. and the length is

$\cancel{5}\left(\frac{12}{\cancel{5}}\right) = 12$ yds.

Check: The area is $\frac{12}{5} \cdot 12 = \frac{144}{5} = 28\frac{4}{5}$. The perimeter is $2\frac{2}{5} + 2\frac{2}{5} + 12 + 12 = 28\frac{4}{5}$.

**For more help:**

Go to **www.khanacademy.org/math/algebra**
Click on "Polynomials " under Topics and look at
Solving a quadratic by factoring

KHAN ACADEMY

**End of Lesson 4**