Algebra IA

Unit 1 Lesson 7/8 Classroom Extension

Addition/Subtraction of Signed (+/-) Numbers (Integers)

Objectives

- Understand how to add and subtract with manipulatives.

Instructional Materials

- Two different colored groups of items. Suggestions:
    - Colored paper strips
    - Plastic chips
    - Pennies & nickels
    - buttons
- Combining integers worksheet (Next Page)

Classroom Activity

- Let one color represent positive 1, and the other color negative 1.
- Establish the rule that the combination of a positive integer chip and a negative integer chip is zero. In other words, when you place a positive and negative next to each other, they cancel out.
- Provide students with 5 of each color and have them model the simple integer operations provided on the activity sheet.
- Make sure students represent addition as adding positives or negatives, and subtraction as taking away positives or negatives.

Other Content Area Tie-in

- Integers often come up when discussing temperature and money. Also, multiplication and division are derived from addition and subtraction.

Combining Integers

Name:______________________________

Separate your items into two groups or categories (color is most common).

Categories:

Let one group represent positive integer chips. Let the other represent negative integer chips.

(Category)

Positive Integer: ________________ (Color or shape)

Negative Integer: ________________ (Color or shape)

Now that we have our positive and negative *integer chips*, please model the following problems. Then, draw a picture of your model in the space provided. (Refer back to the examples in lesson 3 for help)

1. $5 + 3$
2. $6 - 2$
3. $-2 + 3$
4. $-5 - 1$
5. $7 - (-2)$
6. $5 + (-5)$

Create your own integer problems, and share with your group.

Unit 1 Lesson 10/11 Classroom Extension

Fractions and Number Sense/Operation with Fraction

Objective

- ⇨ Use manipulatives to add and compare fractions with unlike denominators
- ⇨ Use manipulatives to grasp the concept of equivalent forms of fractions

Instructional Materials

- ⇨ Copies of the activity sheets
- ⇨ Copies of the handout with the shapes to be cut out
- ⇨ Scissors

Classroom Activity

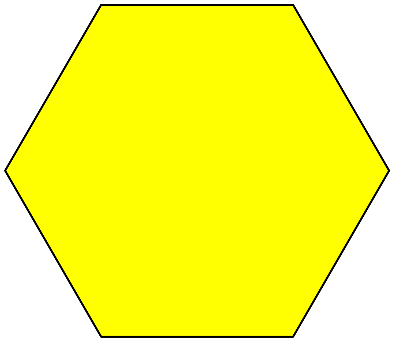
- ⇨ Visit http://illuminations.nctm.org/LessonDetail.aspx?ID=U113 and follow instructions listed at the top.
- ⇨ Provide each student with a copy of the attached worksheet and ask them to model the problems with the cut out shapes provided.
- ⇨ Have them model various addition problems using the colored shapes.

Other Content Area Tie-in

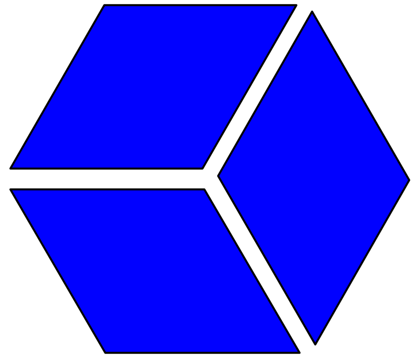
- ⇨ This will easily tie into equivalent fractions. It will show them that two triangles make a parallelogram, three triangles make a trapezoid. This shows them what equivalent fractions can be. Using more than one whole can also form mixed numbers.

Please cut out the following shapes

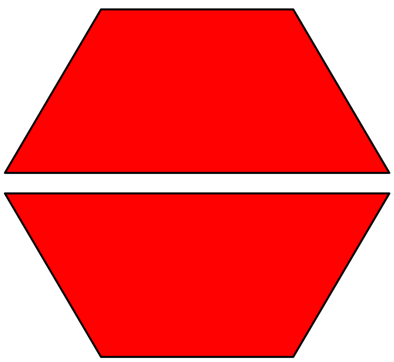
1 whole =

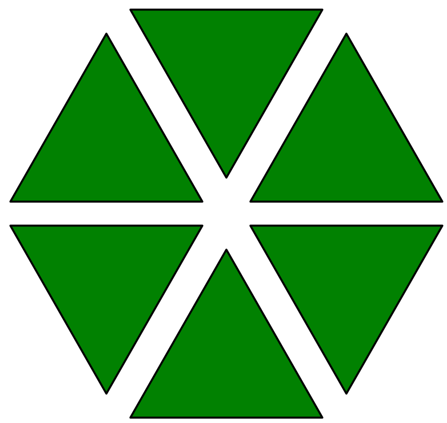


$\frac{1}{3}$ =



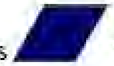
$\frac{1}{2}$ =





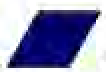
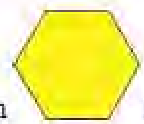
Region Relationships

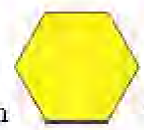
NAME ________________________

1. How many green triangles  are in one blue rhombus  ?

2. How many green triangles  are in one red trapezoid  ?

3. How many green triangles  are in one yellow hexagon  ?

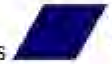
4. How many blue rhombuses  are in one yellow hexagon  ?

5. How many red trapezoids  are in one yellow hexagon  ?

Region Relationships

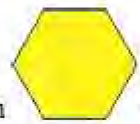
NAME ____________________

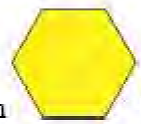
1. How many green triangles  are in one blue rhombus  ?

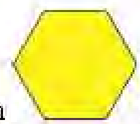
   The green triangle  is what fraction of the blue rhombus  ?

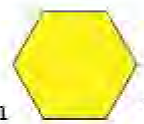
2. How many green triangles  are in one red trapezoid  ?

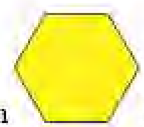
   The green triangle  is what fraction of the red trapezoid  ?

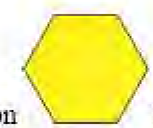
3. How many green triangles  are in one yellow hexagon  ?

   The green triangle  is what fraction of the yellow hexagon  ?

4. How many blue rhombuses  are in one yellow hexagon  ?

   The blue rhombus  is what fraction of the yellow hexagon  ?

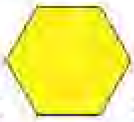
5. How many red trapezoids  are in one yellow hexagon  ?

   The red trapezoid  is what fraction of the yellow hexagon  ?

Region Relationships

NAME ________________________

Use two yellow hexagons as the whole.

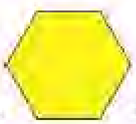
1. The yellow hexagon  is what part of the whole?

2. The red trapezoid  is what part of the whole?

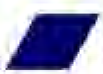
3. The blue rhombus  is what part of the whole?

4. The green triangle  is what part of the whole?

Use three yellow hexagons as the whole.

1. The yellow hexagon  is what part of the whole?

2. The red trapezoid  is what part of the whole?

3. The blue rhombus  is what part of the whole?

4. The green triangle  is what part of the whole?

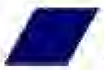
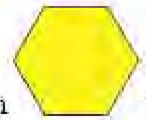
Region Relationships

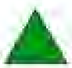
NAME ______________________

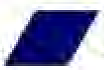
1. The green triangle  is what fraction of the blue rhombus  ?

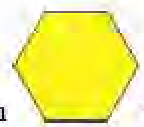
   The red trapezoid  is what fraction of the yellow hexagon  ?

   How can  and  both be equivalent to $\frac{1}{2}$?

2. The blue trapezoid  is what fraction of the yellow hexagon  ?

   The green triangle  is what fraction of the red trapezoid  ?

   How can  and  both be equivalent to $\frac{1}{3}$?

3. The green triangle  is what fraction of the yellow hexagon  ?

   What pattern block represents $\frac{1}{6}$ when you use two yellow hexagons as the whole?

   How can two different pattern blocks both be equivalent to $\frac{1}{6}$?

Unit 2 Lesson 5 Classroom Extension
Absolute Value

Objective

- ⇨ Investigate how absolute value is used in everyday life.

Instructional Materials

- ⇨ Newspapers

Classroom Activity

- ⇨ Analyze several days of reports from the newspaper that show the high and low temperatures for cities across the country.
- ⇨ Assign each student a city in which they will record the rise and fall of the days' high temperatures on a vertical number line.
- ⇨ Observe and contrast the differences between the temperatures. Discuss how these differences relate to absolute value.

Other Content Area Tie-in

- ⇨ Absolute value is used in statistical problems, such as finance and medicine. Suppose you are driving a car. Driving too fast is a hazard and may earn a speeding ticket. Driving too slow is also a hazard, and may also result in being ticketed. What matters is how different one's speed is from the speed limit. This type of "difference" is fundamental to all sorts of concepts in statistics, where the absolute value is used in various ways of quantifying how well or how poorly one thing predicts another.

Unit 2 Lesson 6 Classroom Extension
Order of Operations

Objectives

- ⇨ Use order of operations to simplify numerical expressions.
- ⇨ Develop algebraic understanding as expressions are simplified.

Instructional Materials

- ⇨ Who is Correct? Sheet (refer to NCTM's website listed below)
- ⇨ Questions for students:

  Why is it important for everyone to follow the same order in simplifying expressions?
  [If we didn't, we would come up with various answers for the same numerical expression.]

Classroom Activity

- ⇨ Visit http://illuminations.nctm.org/LessonDetail.aspx?ID=L643 (NCTM). Follow the instructional plan.

Math Connections

- ⇨ This makes connections with reading math. Students should improve problem solving strategies. This lesson forces students to work together to reason through the problem. This problem can force students to evaluate and explain why their answer is correct.

Unit 2 Lesson 12-14 Classroom Extension
A Visual Representation of Logic

Objectives

- ⇨ Use pictures to develop an understanding of conditional statements
- ⇨ Give examples of direct, indirect, and transitive reasoning
- ⇨ Identify valid and invalid arguments

Instructional Materials

- ⇨ Direct, Indirect, and Transitive Reasoning Worksheets
- ⇨ Valid or Invalid Arguments Worksheet
- ⇨ Reference Sheet

Classroom Activity

- ⇨ Visit http://illuminations.nctm.org/LessonDetail.aspx?id=L384 (NCTM). Follow the instructional plan.

Math Connections

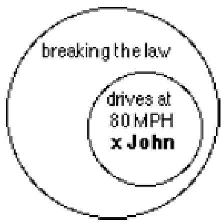
- ⇨ This makes connections with representing ideas visually. It assists the students in realizing how beneficial visual models can be in solving various types of problems. This should help students understand the notation of logic symbols as well

Direct Reasoning

NAME ___________________________

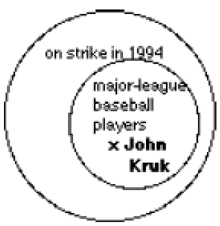
For questions 1-3, look at the picture and write two forms of the argument.

Example:

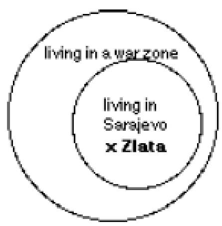


Premise:	Everyone who drives at 80 MPH is breaking the law.	$p \to q$:	If you drive at 80 MPH, then you are breaking the law.
Premise:	John is driving at 80 MPH.	p:	John is driving at 80 MPH.
Conclusion:	John is breaking the law.	q:	John is breaking the law.

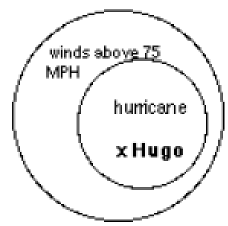
1.



2.



3.



For questions 4–6, draw the proper conclusion and give the corresponding diagram.

4. If the density (in g/cm^3) of a substance is less than 1, then the substance will float. The density of oak is between 0.6 and 0.9. We can thus conclude that ___________________________.

5. Classical Doric architecture used symmetry for aesthetic reasons. the Basilica at Paestum in southern Italy is a Doric temple. We can thus conclude that ___________________________.

6. If a, b, and c are real numbers, then $a(b + c) = ab + ac$. These three numbers are real numbers: 2.1, -8, and 17. We can thus conclude that ________________________.

7. In *Hound of the Baskervilles*, Sherlock Holmes says to James Mortimer, "I observe from the yellow stain on the forefinger that you make your own cigarettes." Write a valid argument that has Holmes's deduction as a conclusion. Give an appropriate diagram.

9. Explain why the following argument is invalid. Draw the corresponding diagram.

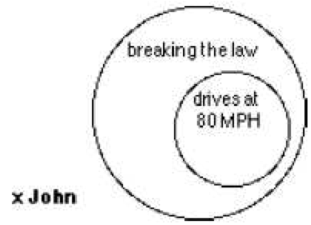
Premise:	All plastic toys are unbreakable.
Premise:	This yellow truck is unbreakable.
Conclusion:	This yellow truck is plastic.

Indirect Reasoning

NAME ______________________________

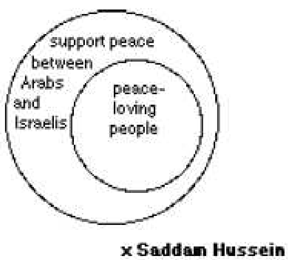
For questions 1-3, look at the picture and write two forms of the argument.

Example:

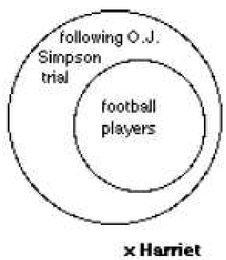


Premise:	Everyone who drives at 80 MPH is breaking the law.	$p \rightarrow q$:	If you drive at 80 MPH, then you are breaking the law.
Premise:	John is not breaking the law.	p:	John is not breaking the law.
Conclusion:	John is not driving 80 MPH.	q:	John is not driving 80 MPH.

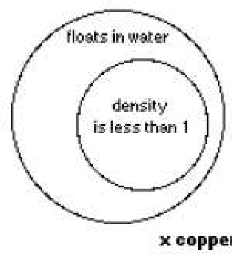
1.



2.



3.



For questions 4–6, draw the proper conclusion and give the corresponding diagram.

4. Everyone who studies chaos theory is interested in dynamical systems. Richard Gere has no interest in dynamical systems. We can thus conclude that _________________.

5. If the operation is addition or multiplication, then the associative law holds. For the operation given, the associative law does not hold. We can thus conclude that ____________________.

6. If you live in Kobe, Japan, then you live in a quake-stricken city. Toshima does not live in a quake-stricken city. We can thus conclude that __________________________.

7. On examining the room, Sherlock Holmes observes, "Something has been taken. There is not as much dust in this corner of the shelf than elsewhere." Write a valid argument that uses indirect reasoning and has Holmes's deduction as its conclusion. Draw an appropriate diagram.

8. Explain why the following argument is invalid. Draw the corresponding diagram.

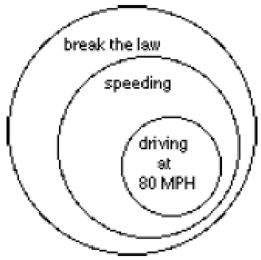
Premise:	All plastic toys are unbreakable.
Premise:	This yellow truck is not plastic.
Conclusion:	This yellow truck is not unbreakable.

Transitive Reasoning

NAME ____________________________

For questions 1-3, look at the picture and write two forms of the argument.

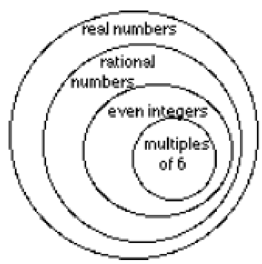
Example:



Premise:	Everyone who drives at 80 MPH is speeding.	$p \rightarrow q$:	If you drive at 80 MPH, then you are speeding.
Premise:	All who speed break the law.	p:	If you speed then you break the law.
Conclusion:	Everyone who drives at 80 MPH breaks the law.	q:	If you drive at 80 MPH, then you break the law.

1.

2.



3.

For questions 4–5, draw the proper conclusion and give the corresponding diagram.

4. All rational numbers are real numbers. All real numbers are complex numbers. We can thus conclude that ________________________.

5. If you voted for Oregon's "Measure 16," then you support euthanasia. If you support euthanasia, then you believe in some form of doctor-assisted suicide. We can thus conclude that ______________________.

6. Saying that some elements of A are in B is the same as saying $A \cap B$. If $A \cap B$ and $B \cap C$, can you conclude that $A \cap C$? Draw a picture to show that this conclusion could be false.

7. Explain why the following argument is invalid. Draw the corresponding diagram.

Premise:	Some plastic toys are yellow.
Premise:	Some yellow toys are on the floor.
Conclusion:	Some plastic toys are on the floor.

Valid or Invalid

NAME ______________________________

For each of the following examples, decide if the argument is valid or invalid, and draw a Venn diagram that justifies your response. If the argument is valid, give the type of reasoning used.

1. If someone buys a new Lamborghini, she or he will pay over $200,000.
Marie does not buy a new Lamborghini.
Therefore, Marie does not pay over $200,000 for her new car.

2. If people listen to Rush Limbaugh, they will hear a lot of gossip about Washington.
Mr. Jones hears gossip about Washington all the time.
Mr. Jones must listen to Rush Limbaugh.

3. In Algeria, if the terrorists believe that someone is promoting Western culture, he or she is in danger of being assassinated.
The terrorists believe that all barbers are promoting Western culture.
In Algeria, all barbers are in danger of being assassinated.

4. In China, job applicants do not ask how much they will be paid when they are hired.
When Jin Tai was hired, he asked his employer how much he would be paid.
Jin Tai must have been hired outside of China.

5. If Alec washes the school's windows, he will be paid $5.00 an hour.
Alec washes the school's windows for four hours, so he gets paid $20.00.

6. If Maria leaves work at five o'clock, she will run into rush-hour traffic.
If Maria runs into rush-hour traffic, she will arrive home in a bad mood.
Therefore, if Maria leaves work at five o'clock, she will arrive home in a bad mood.

7. Some solutions to the equations are integers.
Some integers are less than zero.
We can conclude that some solutions to the equation are less than zero.

8. If a politician decides to run for president, then he or she will make many visits to New Hampshire.
Senator Dole has decided to run for president.
Senator Dole will make many visits to New Hampshire.

9. Five identical sweatshirts are placed in a bag. A letter is stitched to the back of each shirt; two of the letters are L's and three are W's. Chris, Hugo, and Mary each pull out a shirt without looking at it and put it on. Chris can see Mary's and Hugo's shirts and correctly deduces, "I cannot tell which letter I have on." Mary sees only Hugo's shirt and draws the same valid conclusion. Hugo sees no one's shirt but uses his logic and is able to tell which letter is on his back. How does Hugo do it? First write a valid argument that involves Chris's deduction. Then using that conclusion, write a second argument to justify Hugo's claim. Diagram your arguments.

Reference Sheet

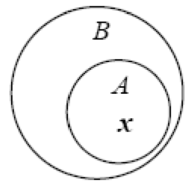
The following symbols are used on this sheet:

$\subset$	$\in$	$\notin$	$\cap$
"is a subset of"	"is an element of"	"is not an element of"	"intersects"

Direct Reasoning

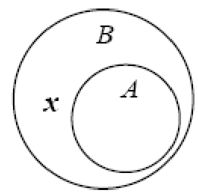
THE UNDERLYING IDEA

If $A \subset B$ and $x \in A$, then we can conclude that $x \in B$.



Premise:	All elements in A are also in B.	$p \rightarrow q$:	If an element is in A, then it is in B.
Premise:	x is an element in A.	p:	x is an element in A.
Conclusion:	x is an element in B.	$\therefore q$:	x is in B.

The argument is VALID because if A is in B and x is in A, then we can be 100 percent certain that x is in B.

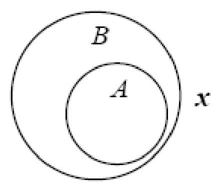


But watch out! If $A \subset B$ and $x \in B$, we cannot be 100 percent certain that $x \in A$. The argument would be invalid. The diagram to the right shows that $A \subset B$ and $x \in B$, but $x \notin A$.

Indirect Reasoning

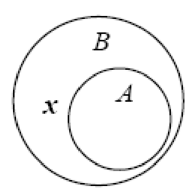
THE UNDERLYING IDEA

If $A \subset B$ and $x \notin B$, then we can conclude that $x \notin A$.



Premise:	All elements in A are also in B.	$p \rightarrow q$:	If an element is not in B, then it is not in A.
Premise:	x is not an element in B.	p:	x is not an element in B.
Conclusion:	x is not an element in A.	$\therefore q$:	x is not in A.

The argument is VALID because if A is in B and x is not in B, then we can be 100 percent certain that x is not in A.

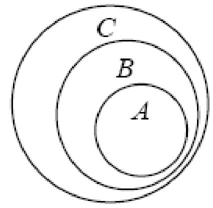


But watch out! If $A \subset B$ and $x \notin A$, we cannot be 100 percent certain that $x \notin B$. The argument would be invalid. The diagram to the right shows $A \subset B$ and $x \notin B$, but $x \notin A$.

Transitive Reasoning

THE UNDERLYING IDEA

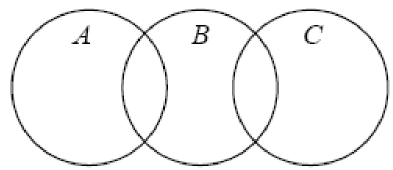
If $A \subset B$ and $B \subset C$, then we can conclude that $A \subset C$.



Premise:	All elements in A are also in B.	$p \rightarrow q$:	If an element is not in B, then it is not in A.
Premise:	x is not an element in B.	p:	x is not an element in B.
Conclusion:	x is not an element in A.	$\therefore q$:	x is not in A.

The argument is VALID because if A is in B and B is in C, then we can be 100 percent certain that A is in C.

But watch out! The diagram to the right shows that if $A \cap B$ and $B \cap C$, it is not necessarily true that $A \cap C$.



Unit 3 Lesson 5 Classroom Extension
Combining Like Terms

Objective

- Use manipulatives to recognize and combine like terms

Instructional Materials

- Candy manipulatives (such as Skittles or M&Ms)

Classroom Activity

- Provide each student with a small bag of candy. Assign a variable, algebraic term, or constant to each color. (For example, let red represent x, let green represent y, let blue represent x^2, let yellow represent 1, etc.)
- Describe each collection of candy by collecting like terms.
- Use a black or whiteboard to record each student's algebraic expression that represents their candy. Then have students combine like terms to represent the total collection of candy.

Other Content Area Tie-in

- Discuss how like terms are used in every day life. (Example: Each person in the classroom can share their age and then the "like ages" can be combined.)

Unit 3 Lesson 10-14 Classroom Extension

Multiplying/Dividing Polynomials

Objectives

- Understand the relationship between expanding and factoring polynomials
- Factor trinomials
- Multiply monomials and binomials

Instructional Materials

- Polynomial Puzzler Overhead
- Polynomials Puzzler Activity Sheet (And Answer Key)

Classroom Activity

- Visit http://illuminations.nctm.org/LessonDetail.aspx?id=L798 (NCTM). Follow the instructional plan.
- Questions for students:
    - Did you try to expand first, and factor only if the spaces couldn't be fill in otherwise? Did you seek out the spaces that required factoring first?
    - Did you use a traditional method to expand and factor, such as FOIL, or did you develop your own strategies as you worked?
    - What is the mathematical relationship between expanding and factoring?
        - Expanding and factoring are inverses of one another. Students may also talk about the fact that expanding is multiplying and factoring is dividing.

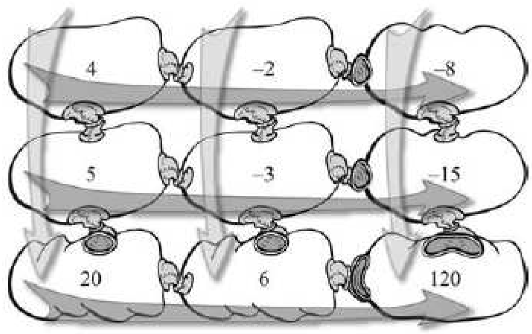
Math Connections

- The students will make connections between multiplying and dividing polynomials. Students will be able to make the connection between factors of a polynomials and dividing polynomials.

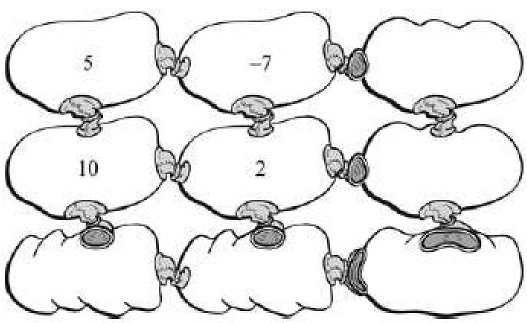
OVERHEAD

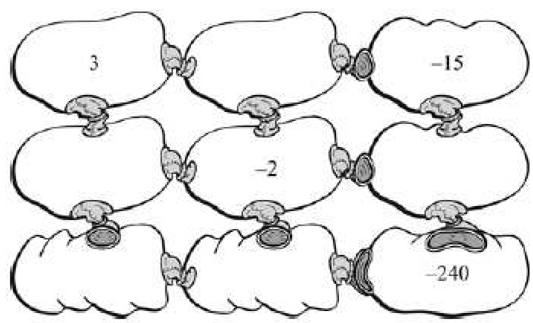
Polynomial Puzzler

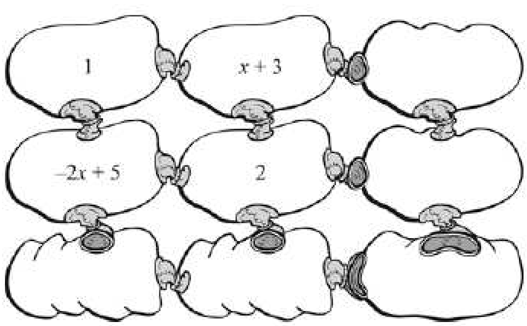
Each arrow below indicates multiplication.
For example, $4 \times 5 = 20$ and $5 \times -3 = -15$.

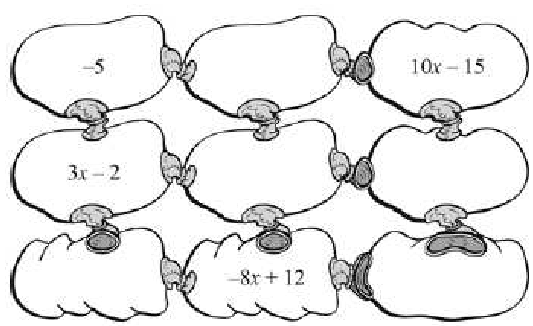


Now try filling in the empty spaces for these:

1.


2.


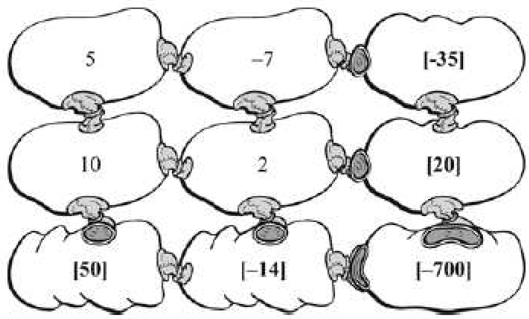
3.


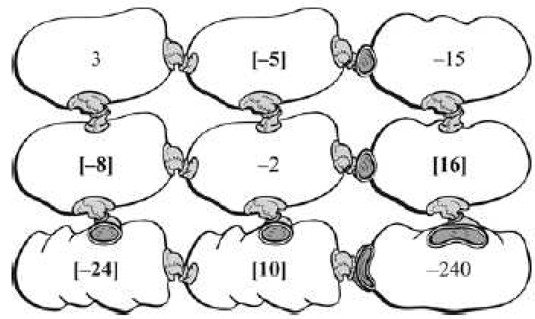
4.


OVERHEAD

Polynomial Puzzler

Answers:

1.


2.


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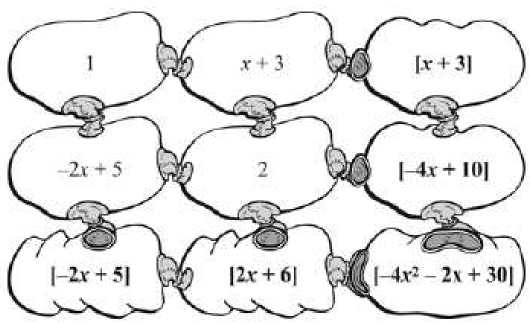

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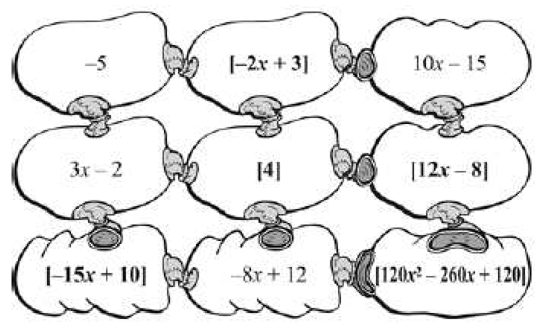
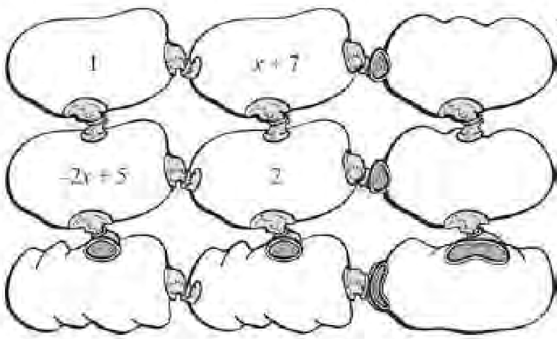


Polynomial Puzzler

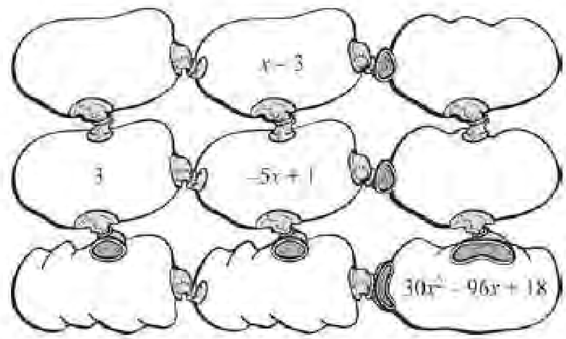
NAME__________________________

Fill in the empty spaces to complete the puzzle. In any row, the two left spaces should multiply to equal the right-hand space. In any column, the two top spaces should multiply to equal the bottom space.

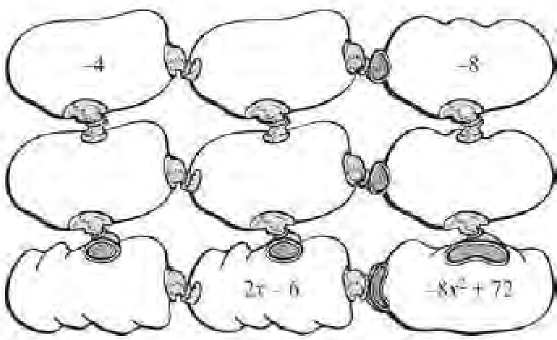
1.



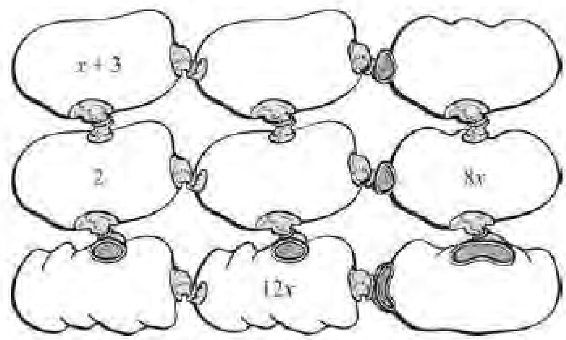
2.



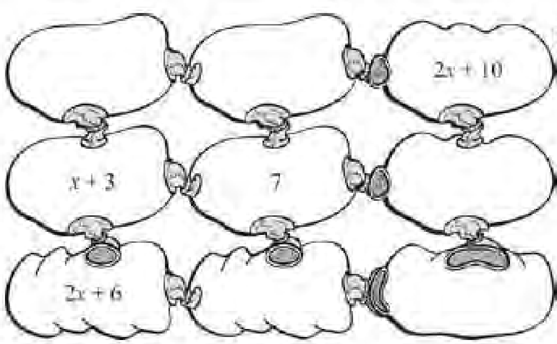
3.



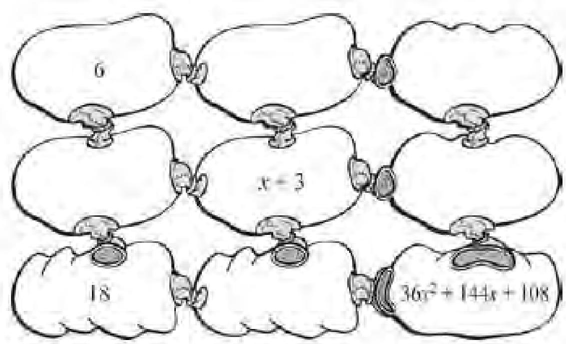
4.



5.



6.



Answer Key – Polynomial Puzzler

Fill in the empty spaces to complete the puzzle. In any row, the two left spaces should multiply to equal the right-hand space. In any column, the two top spaces should multiply to equal the bottom space.

1.

1	$x + 7$	$[x + 7]$
$-2x + 5$	2	$[-4x + 10]$
$[-2x + 5]$	$[2x + 14]$	$[-4x^2 - 18x + 70]$

2.

$[-2]$	$x - 3$	$[-2x + 6]$
3	$-5x + 1$	$[-15x + 3]$
$[-6]$	$[-5x^2 + 16x - 3]$	$30x^2 - 96x - 18$

3.

-4	$[2]$	-8
$[x + 3]$	$[x - 3]$	$[x^2 - 9]$
$[-4x - 12]$	$2x - 6$	$-8x^2 + 72$

4.

$x + 3$	$[3]$	$[3x + 9]$
2	$[4x]$	$8x$
$[2x + 6]$	$12x$	$[24x^2 + 72x]$

5.

$[2]$	$[x - 5]$	$2x + 10$
$x + 3$	7	$[7x + 21]$
$2x + 6$	$[7x - 35]$	$[14x^2 + 112x + 210]$

6.

6	$[2x + 2]$	$[12x + 12]$
$[3]$	$x + 3$	$[3x + 9]$
18	$[2x^2 + 8x + 6]$	$36x^2 + 144x + 108$

Unit 4 Lesson 2 Classroom Extension
The Difference of Two Squares

Objectives

- ⇨ Analyze and represent patterns using algebraic symbols
- ⇨ Represent and compare quantities with integers
- ⇨ Communicate mathematical language clearly

Instructional Materials

- ⇨ Pen/Pencil and paper

Classroom Activity

- ⇨ Ask each student to do the first three steps individually and without sharing with their neighbors. Step 4 will be discussed after some experimenting with various numbers.
    - ⇨ Step 1: Pick any two consecutive integers
      (Consecutive integers occur one right after the other {…3, 4, 5, 6 …})
    - ⇨ Step 2: Square each number and find the difference
    - ⇨ Step 3: Find the sum of the two original numbers
    - ⇨ Step 4: Explain why steps two and three give the same result
- ⇨ One possible reaction when students complete Steps 2 and 3 is often amazement, followed by confusion. Allowing students to communicate with fellow students is an invaluable tool. Students who are comfortable with variables and symbolic manipulation usually solve the problem using an appropriate algebraic equation, for example,
    - $\mathbf{(n+1)^2 - n^2 = (n^2 + 2n + 1) - n^2 = 2n + 1 = (n + 1) + n}$

  (It’s important to realize that if n is the first number, $n + 1$ is always the second number.)
- ⇨ A solution such as that in the above figure can set the stage for a discussion of the power of algebraic symbols; however, you may wish to delay this discussion while other students explore the problem using a more arithmetic approach, one that is also at the heart of algebra as generalized arithmetic. This approach usually involves repeating steps 1-3 for several pairs of consecutive numbers.

⇨ The figure below shows a sample of the results of this approach after the students have organized their data. Invariably, several students try to find a pattern in the various computations that they used to test the validity of Step 4. The students often have some interesting observations, but they usually cannot adequately explain the underlying reasons for the similarities in the results of the computations contained in Steps 2 and 3.

$4^2 - 3^2 = 16 - 9 = \mathbf{7}$	$4 + 3 = \mathbf{7}$
$5^2 - 4^2 = 25 - 16 = \mathbf{9}$	$5 + 4 = \mathbf{9}$
$6^2 - 5^2 = 36 - 25 = \mathbf{11}$	$6 + 5 = \mathbf{11}$
$10^2 - 9^2 = 100 - 81 = \mathbf{19}$	$10 + 9 = \mathbf{19}$

⇨ Because further investigation of patterns may be necessary for these students, pose a similar task which is designed to supply scaffolding for the students' thinking. However, instead of using two consecutive numbers, use two numbers that differ by 2 (e.g., 4 and 6). Their investigations may lead to information similar to that in the previous figure. If necessary, repeat this process for numbers that differ by 3, but many students will attempt this exploration on their own.

$4^2 - 2^2 = 16 - 4 = \mathbf{12}$	$4 + 2 = \mathbf{6}$	$\mathbf{12} \div 2 = \mathbf{6}$
$5^2 - 3^2 = 25 - 9 = \mathbf{16}$	$5 + 3 = \mathbf{8}$	$\mathbf{16} \div 2 = \mathbf{8}$
$6^2 - 4^2 = 36 - 16 = \mathbf{20}$	$6 + 4 = \mathbf{10}$	$\mathbf{20} \div 2 = \mathbf{10}$

⇨ At this stage, the students are in a position to cross the algebraic divide. Encourage a discussion in which students examine a holistic view of all the computations and attend to the patterns found in them. Ask students what specific observations they made that allowed them to express the general rule. In response, such comments may include "The second one was the sum of the numbers times 2, so I guessed that the third one is the sum of the numbers times 3."

⇨ The discussion can continue using such language, and the students can discover the general pattern in the computation. At times, students may invent their own language to discuss their discoveries. Students may use the notion of "start-off numbers" to explain how they understood the general computational aspects of the difference-of-squares factoring pattern.

- ⇨ Although algebraic understanding has emerged, the need to express the ideas with formal algebraic symbols may still exist. If so, ask the students to use variables to write the results of the three investigations. Their equations for each investigation, respectively, may resemble the following:

Case 1: When $b = a - 1$, then $a^2 - b^2 = a + b$

Case 2: When $b = a - 2$, then $a^2 - b^2 = 2(a + b)$

Case 3: When $b = a - 3$, then $a^2 - b^2 = 3(a + b)$

- ⇨ Appropriate discussion or student insight can guide students to realize that the coefficient outside the parentheses in those three cases is equal to the difference between *a* and *b*. That is, $a - b$ is equal to 1, 2, and 3, respectively. Eventually, this may lead students to form a conjecture about the general factoring pattern:

$$a^2 - b^2 = (a + b)(a - b)$$

- ⇨ Slowly and with guidance, the students make an algebraic leap. Further, they require very little knowledge of symbolic manipulation to produce the expression; the construction is grounded in their arithmetic experiences.

Math Connections

- ⇨ Connections will be made between patterns and algebraic expressions. Students will learn how to model patterns using algebra, as well as learn of the significance of representing problems using variables.

Unit 5 Lesson 12 Classroom Extension

Work Problems and Percent Problems

Objectives

- ⇨ Recognize equivalent ratios.
- ⇨ Solve proportions to estimate population size.

Instructional Materials

- ⇨ Paper cups
- ⇨ White beans
- ⇨ Marker
- ⇨ Capture-Recapture Activity Sheet (refer to NCTM's website listed below)

Classroom Activity

- ⇨ Visit http://illuminations.nctm.org/LessonDetail.aspx?id=L721 (NCTM). Follow the instructional plan.

Other Content Area Tie-in

- ⇨ The Questions for Students on the Capture-Recapture website (NCTM) provide an analysis of how proportions relate to the population of robins.

Algebra IB

Unit 1 Lesson 1 Classroom Extension

Functions and Relations

Objectives

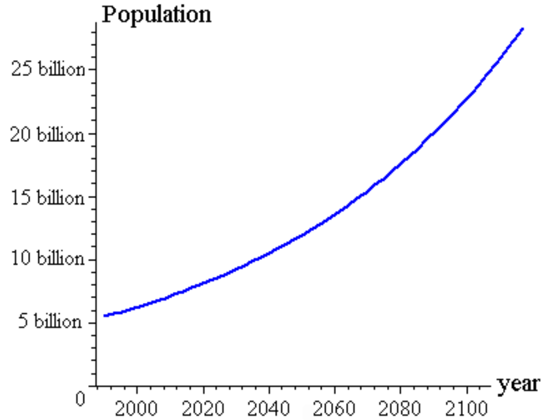
- ⇨ Distinguish between relations and functions.
- ⇨ Create a model of a function.

Classroom Activity

- ⇨ Consider a soda machine that charges $1.00 per soda and offers Soda A, Soda B, Soda C, Soda D, and Soda E. Does this soda machine represent a function? (No since the same element in the domain [$1.00] is assigned to five different elements in the range [Soda A, Soda B, Soda C, Soda D, and Soda E].)
- ⇨ Design a soda machine that is a function. Provide an illustration of the machine and explain why it is a function. (Example: Let the cost of each soda be unique.)

Math Connections

- ⇨ The population of the earth is growing at approximately 1.3% per year. The graph below shows the relationship between time and population. This relation is an example of a function since each element of the domain (years) is paired with exactly one element of the range (population).



Unit 1 Lesson 5 Classroom Extension

Slope of a Line

Objectives

- Investigate how slope relates to real-world data.
- Use slope to approximate rates of change in the height of boys and girls at different ages.

Instructional Materials

- Rate of Change Activity Sheet (refer to NCTM's website listed below)

Classroom Activity

- Visit http://illuminations.nctm.org/LessonDetail.aspx?id=L668 (NCTM). Follow the instructional plan.

Other Content Area Tie-in

- Discuss other areas in which slope may be used to represent a rate of change. (Example: Gas mileage in miles/hour)

Unit 1 Lesson 8 Classroom Extension
Effects of Change of Slope and Intercepts

Objectives

- Graph a line based on data points
- Find the equation of a line
- Identify the *y*-intercept and slope, and state their significance in the context of the problem

Instructional Materials

- Rate of Change Activity Sheet (refer to NCTM's website listed below)

Classroom Activity

- Visit http://illuminations.nctm.org/LessonDetail.aspx?id=L668 (NCTM). Follow the instructional plan.

Other Content Area Tie-in

- Discuss other areas in which slope may be used to represent a rate of change. (Example: Gas mileage in miles/hour)

Unit 1 Lesson 14 Classroom Extension
Applications

Objectives

- ⇨ Investigate scatter plots and the line of best fit using technology.
- ⇨ Use a scatter plot to illustrate data on wildfires.

Instructional Materials

- ⇨ Computers or laptops with internet access

Classroom Activity

- ⇨ Visit http://illuminations.nctm.org/ActivityDetail.aspx?ID=82 (NCTM) and read the instructions found at the top of the page.
- ⇨ Complete the "Exploration" activity.

Other Content Area Tie-in

- ⇨ Visit http://illuminations.nctm.org/LessonDetail.aspx?ID=L663 (NCTM) to investigate wildfires and the correlations between variables using scatter plots and the line of best fit.

Unit 3 Lesson 9 Classroom Extension
Graphing Quadratic Equations

Objectives

- ⇨ Explain the relationship between linear factors of a polynomial function and the graph of the function
- ⇨ Based on the graph of two lines, graph the parabola that is the product of the two lines
- ⇨ Given the graph of a polynomial, find the equations of the line that could be components of the polynomial

Instructional Materials

- ⇨ Student Activity Sheets (Refer to NCTM's website listed below)
- ⇨ Three different colors to write with
- ⇨ Strip of paper or ruler
- ⇨ Graphing calculator (optional)

Classroom Activity

- ⇨ Visit http://illuminations.nctm.org/LessonDetail.aspx?id=L282 (NCTM) and read the instructions found at the top of the page.

Other Content Area Tie-in

- ⇨ Students will make connections between the degree of a function and the way it looks when graphed. Student will also assess how the roots of a quadratic equation affect its graph.

Unit 3 Lesson 14 Classroom Extension
Modeling Exponential Growth and Decay

Objectives

- ⇨ learn to analyze data to determine the type of function that most closely matches the data
- ⇨ demonstrate an understanding of how modifying parameters will change the graphs of functions by writing equations for those functions

Instructional Materials

- ⇨ Student Activity Sheets (Refer to NCTM's website listed below)
- ⇨ M&M's
- ⇨ Paper
- ⇨ Plates, cups, or other containers
- ⇨ Stopwatches
- ⇨ Birthday candles
- ⇨ Matches
- ⇨ Heat-resistant tiles or plates
- ⇨ Rulers
- ⇨ Centimeter grid paper
- ⇨ Weather data
- ⇨ Metersticks
- ⇨ Jug or coffee pot with a spigot
- ⇨ Scissors
- ⇨ Graphing calculators
- ⇨ Strip of paper or ruler
- ⇨ Graphing calculator (optional)

Classroom Activity

- ⇨ Visit http://illuminations.nctm.org/LessonDetail.aspx?id=L300 (NCTM) and read the instructions found at the top of the page.

Unit 4 Lesson 1 Classroom Extension

Systems of Equations – Graphing

Objectives

- ⇨ Create a map of a local town using a system of equations.
- ⇨ Write linear equations to represent roads on the map.
- ⇨ Find the solution to a system of equations graphically.

Instructional Materials

- ⇨ Graph paper
- ⇨ Rulers or straightedges
- ⇨ Colored pencils

Classroom Activity

- ⇨ Follow the steps below to construct a map of a local town.
  1. Using graph paper and a straight edge, draw at least three roads in the town. (Estimate their location.)
  2. Write a linear equation to represent each road on the map.
  3. Color-code the roads and create a key with the equation of each road.
  4. Label the point of intersection of each of the roads.

Other Content Area Tie-in

- ⇨ Examine the map of a city or state and locate the points of intersection of the roads or highways. Discuss why the points of intersection are important when traveling or navigating in the city or state.

Unit 4 Lesson 1-4 Classroom Extension

Systems of Equations

Objectives

- ⇨ Compare two cell phone plans through examples of different usage
- ⇨ Write equations to model allocation of money for each cell phone plan
- ⇨ Solve a system of equations various ways

Instructional Materials

- ⇨ Activity Sheet (Refer to NCTM's website listed below)

Classroom Activity

- ⇨ Visit http://illuminations.nctm.org/LessonDetail.aspx?id=L780 (NCTM) and read the instructions found at the top of the page.
- ⇨ Ask students to solve the system of equations using the various algebraic methods learned in lessons 2 and 3 from this unit.
- ⇨ Ask students the significance of their algebraic solution with respect to the graphical solution.

Other Content Area Tie-in

- ⇨ Students should being to realize the numerous ways to solve a problem. They will make connections between the graph of sets of functions and how it relates to the algebraic definition of the function.

Unit 5 Lesson 5 Classroom Extension
Experimental Probability and Simulations

Objectives

- ⇨ Compare experimental to theoretical probability.

Instructional Materials

- ⇨ Computers or laptops with internet access

Classroom Activity

- ⇨ Visit http://illuminations.nctm.org/ActivityDetail.aspx?ID=79 (NCTM) and read the instructions found at the top of the page.
- ⇨ Complete the "Exploration" activity.

Other Content Area Tie-in

- ⇨ Visit http://illuminations.nctm.org/ActivityDetail.aspx?ID=143 (NCTM) to relate probability to wildfires.

Unit 5 Lesson 11 Classroom Extension
Circle Graphs

Objectives

- ⇨ Compare number of different color M&M's.
- ⇨ Find percentage of different color M&M's.

Instructional Materials

- ⇨ Several vending size bags of M&M's
- ⇨ Paper Plates

Classroom Activity

- ⇨ Every student should wash his/her hands before this activity.
- ⇨ Students will break into groups
- ⇨ Each group will open a bag of M&M's but they cannot eat them yet. They will arrange the M&M's by color on the plate, and count how many of each color.
- ⇨ Students will then organize the colors into a circle on the plate.
- ⇨ Ask students to locate the center of the circle, and draw lines from the center of the circle to the points that separate each color M&M. They should notice that this now looks like a circle graph. Each wedge represents the percent of colors in each individual bag.
- ⇨ Ask students to find the percentages of the colors by dividing the number of each color by the total number of M&M's

Math Connections

- ⇨ Students should make connections between percentages and pie graphs. Students also get practice on collecting data. Students get practice in frequency. This will later connect to area of a section of a circle.

Unit 5 Lesson 14 Classroom Extension
Box-and-Whisker plots

Objectives

- ⇨ Create and read box-and-whisker plots.
- ⇨ Make decisions based on box-and-whisker data collected.

Instructional Materials

- ⇨ Box-and-whisker activity sheet (refer to NCTM's website listed below)
- ⇨ NBA player statistics (refer to NCTM's website listed below)

Classroom Activity

- ⇨ Visit http://illuminations.nctm.org/LessonDetail.aspx?id=L737 (NCTM). Follow the instructional plan.
- ⇨ Ask the students to compare the median and mean of the data

Other Content Area Tie-in

- ⇨ Students will relate this to collecting data. They will also relate it to finding the median, mode, and range. Students should be able to notice how the data spread affects the look of the box-and-whisker plot.

End of Algebra Classroom Extensions